CS 6320, 3D Computer Vision
Spring 2013, Prof. Guido Gerig

Assignment 3: Photometric Stereo & Epiolar Geometry

Out:	Monday 03-04-2013
Due:	Monday 03/25-2013 (theoretical and practical parts)
TA:	Anshul Joshi, MEB 3115
Office hours:	TA: Mo/We 3pm to 5pm, G. Gerig Mo/We 3pm to 5pm

Required Readings: Computer Vision, Forsyth & Ponce, Chapters 4/5 and 10/11
In particular slides to these chapters provided on Course Webpage

Grading

Theoretical problems: These serve as your own study of the material using the textbook and all materials provided on WebCT. Detailed solutions will be provided.

Practical problem: Grading will primarily concern your solution strategy and solution of the camera calibration, and the report that describes your project, your development of the methodology, results, and critical assessments of the results.

1 I. Theoretical Problems

1.1 BDRF

a) Why are specularities brighter than diffuse reflection? (Question 4.9 Forsyth&Ponce, explain in a few sentences), use diagrams.

b) The eye responds to radiance. Explain why Lambertian surfaces are often referred to as having a brightness independent of viewing angle (Question 4.11, explain in a few sentences), use diagrams.

1.2 Reflectance Map

Some surface material cannot be simplified to have Lambertian reflectance properties. The "maria" of the moon, e.g., the dark formation as observed from the earth, is a typical example of a non-Lambertian surface. In this case, we observe that the surface emits radially equally in all directions, or in other words, brightness seems to increase with increasing angle between viewer and surface (hint: remember derivation of the equation for Lambertian surfaces where only the foreshortening of the incoming angle is considered).

a) Derive the equation for the reflectance map R(p,q) in this special case.

b) What are the isodensity curves of constant brightness in R(p,q)? (Hint: remember the discussion of circles and conic sections for the Lambertian case)

c) What are the contours of constant brightness of an image of a sphere?

d) Sketch a solution to estimate (p,q) from multiple images with different light source directions. How many would you need?

1.3 Stereo System

a) Starting from the disparity equation $d = -Bf/Z$, build the relationship between $\triangle Z$ and $\triangle d$. Hint: Build the derivative $\frac{\partial d}{\partial Z}$ and discuss changes in disparity versus changes in depth as a function of depth.

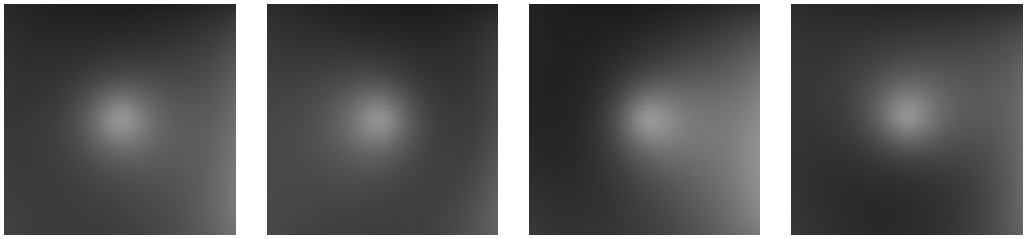
b) Estimate the accuracy of the simple stereo system with two rectified images assuming that the only source of noise is the localization of corresponding points in the two images (the disparity d). (Hint: Take the partial derivatives of Z with respect to d,B,f (disparity, baseline, focal length)). Then use the general formula for error estimation of multiple independent variables as $\partial Z = \sqrt{(\frac{\partial Z}{\delta d}\partial d)^2 + (\frac{\partial Z}{\partial B}\delta B)^2 + (\frac{\partial Z}{\partial f}\delta f)^2}$. Discuss the dependence of the error in depth estimation as a function of the baseline width and the focal length.

c) Using the previous solution, estimate the accuracy with which features should be localized in the two images in order to reconstruct depth with a relative error $|\frac{\delta Z}{Z}|$ smaller than 1%.

2 II. Practical Problems

2.1 Problem: Photometric Stereo / Shape from Shading

The goal of this assignment is to implement an algorithm that reconstructs a surface using the concept of photometric stereo. You can assume a Lambertian reflectance function, but the albedo is unknown and nonconstant in the images. Your program will read multiple images as input along with the light source direction for each image. All data sets for this assignment are provided are provided on WebCt.



im1.png	im2.png	im3.png	im4.png
$\begin{bmatrix}0\\0\\1\end{bmatrix}$	$\begin{bmatrix}0.2\\0\\1\end{bmatrix}$	$\begin{bmatrix}-0.2\\0\\1\end{bmatrix}$	$\begin{bmatrix}0\\0.2\\1\end{bmatrix}$

Figure 1: Synthetic data: Images, file names, light source directions. You should also divide the image intensities by 255 to scale them to [0.0,1.0].

Your program should have two parts:

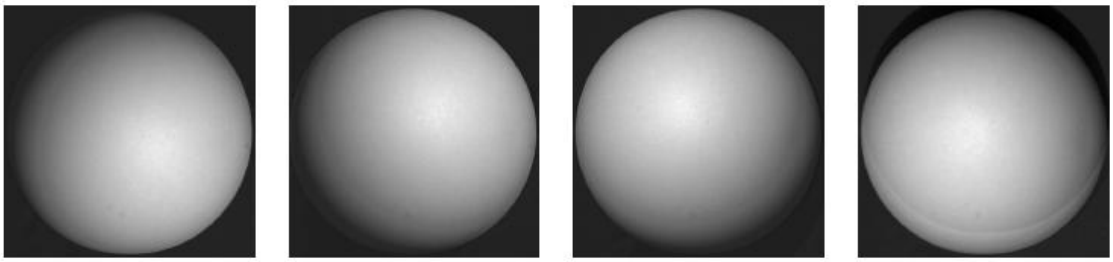
- Read in the images (see Fig. 1), and estimate the surface normals and albedo map.
- Reconstruct the depth map from the normals via integration.

Since we have given you four images (size 100x100), you have the possibility for an exact solution (use of 3 images only) or an overconstrained solution (all four images). The images have intensities in the range 0 to 255, so after reading the images divide them by 255 to get floating point values in the range of $[0 \cdots 1]$.
(Hint: Students reported that using images 1,2, and 4 works ok for the 3 image solution, but that the combination 1,2,3 seems problematic.)
You will have to implement linear least squares in order to use *all four images*, this is not necessary for 3 images. Hint: Follow the F&P book pages 82 and 83 which explains the calculation of the normal and the albedo at each pixel. Matlab strongly supports the solution of equation systems, see help in "Solving Linear System of Equations" in Matlab. Following the book pages 82/83, you will have to solve a system like: $(\mathcal{TV})^t \mathcal{T}i = g(x, y)$. A Matlab notion would be something like : $g = \mathcal{TV} \setminus \mathcal{T}i$;, or $g = pinv(\mathcal{TV}) * \mathcal{T}i$;.
For the second part to get a surface from normals, you should implement the naive direct integration approach to integrate the partial derivaties and construct the depth map (see algorithm 5.1 on page 85 textbook). To simplify integration, you may assume that the object that we are imaging covers the entire image.
For those with additional capacity, there is a link to existing Matlab surface integration software (see below).

2.2 Shape from Shading from real Images

You can choose between the sphere images or the dog images (courtesy to Christopher Bireley, class F2009) or do both once you have a running code.

Sphere Images:



real1.bmp	real2.bmp	real3.bmp	real4.bmp
$\begin{bmatrix} -0.38359 \\ -0.236647 \\ 0.892668 \end{bmatrix}$	$\begin{bmatrix} -0.372825 \\ 0.303914 \\ 0.87672 \end{bmatrix}$	$\begin{bmatrix} 0.250814 \\ 0.34752 \\ 0.903505 \end{bmatrix}$	$\begin{bmatrix} 0.203844 \\ -0.096308 \\ 0.974255 \end{bmatrix}$

Figure 2: Real data: Images, file names, light source directions. You should also divide the image intensities by 255 to scale them from [0,1].

Again, divide the images by 255 to scale them back to a floating point range of $[0 \cdots 1]$. (see Fig. 2).

Dog Images:

dog1.tif	dog2.tif	dog3.tif	dog4.tif
$\begin{bmatrix}16\\19\\30\end{bmatrix}$	$\begin{bmatrix}13\\16\\30\end{bmatrix}$	$\begin{bmatrix}-17\\10.5\\26.5\end{bmatrix}$	$\begin{bmatrix}-9\\25\\4\end{bmatrix}$

Figure 3: Dog image data: Images, file names, light source directions as non-normalized 3D vectors. You should also divide the image intensities by 255 to scale them from [0,1].

Images see Fig. 3. The light source vectors first need to be normalized before using them in your code. Again, divide the images by 255 to scale them back to a floating point range of $[0.0 \cdots 1.0]$. We have got very good results with images 2, 3 and 4, but you can use all 4 images by using SVD for solving for the overconstrained problem.

Integration:

Integration from normals to get the depth map is more difficult for noisy real images since integrability is violated and normals are only estimated approximately. You might first use a single integration step as hown in the slides and in our slides and Forsyth and Ponce book. Alternatively, you might apply integration with different start-points and then average the resulting depth map.

Bonus: Integration with the Chellappa method:

The literature knows more sophisticated reconstructions using calculus of variation, and if you have additional capacity you may try the following:
Matlab code for computer vision, (`http://www.csse.uwa.edu.au/~pk/research/matlabfns/`), under "Surface Normals to Surfaces", surface integration can be done by "frankotchellappa.m" (`http://www.csse.uwa.edu.au/~pk/research/matlabfns/Shapelet/frankotchellappa.m`.

2.3 Report

You should write up a report including your approach and results:

- Estimated albedo map
- Estimated surface normals by either showing:
  - A needle (vector) map (e.g. 2D vectors on the 100x100 grid), or
  - Three images showing the three components of surface normals (normalized normals),
- The reconstructed surface as:

  - Graylevel depth image where depth z is encoded as intensity, and
  - wireframe of a depth map (you can use surf() in matlab) to display Z(X,Y) similarly to figure 5.13 on page 86, and
  - shaded image generated from a user-defined light source position (given a light source direction and surface normals, you can easily calculate shaded images for arbitrary light source positions).

- Commentary about any issues that arose, ways to improve your method, critical assessment of results, etc.