Multi-View Geometry (Chapter 7 and 11 Szelisky)

Guido Gerig

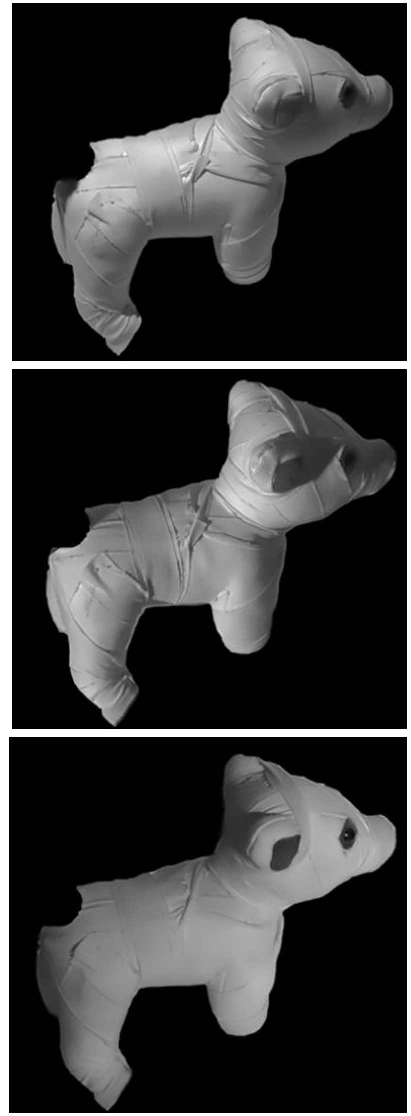
CS 6320 Spring 2012

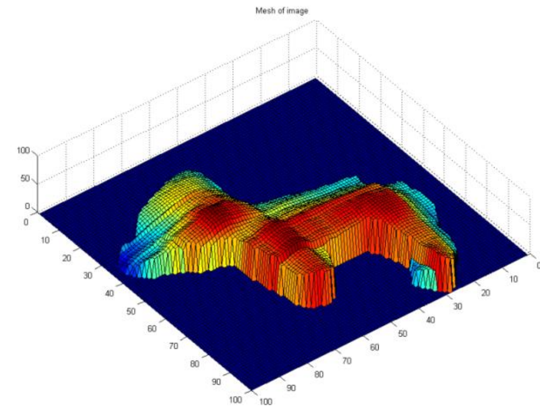
Credits: M. Shah, UCF CAP5415, lecture 23

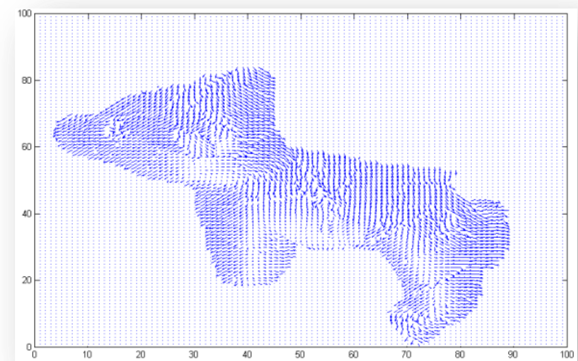
http://www.cs.ucf.edu/courses/cap6411/cap5415/, Trevor Darrell, Berkeley, C280, Marc Pollefeys

Visual cues

- Shading

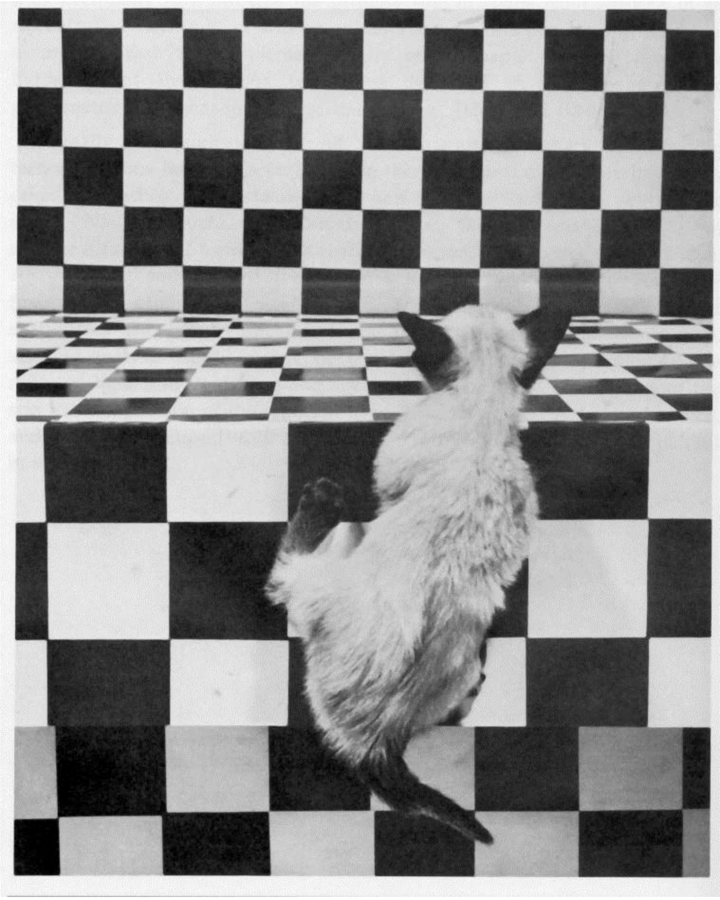






Visual cues

- Shading
- Texture



***The Visual Cliff*, by William Vandivert, 1960**

Visual cues

- Shading
- Texture
- Focus

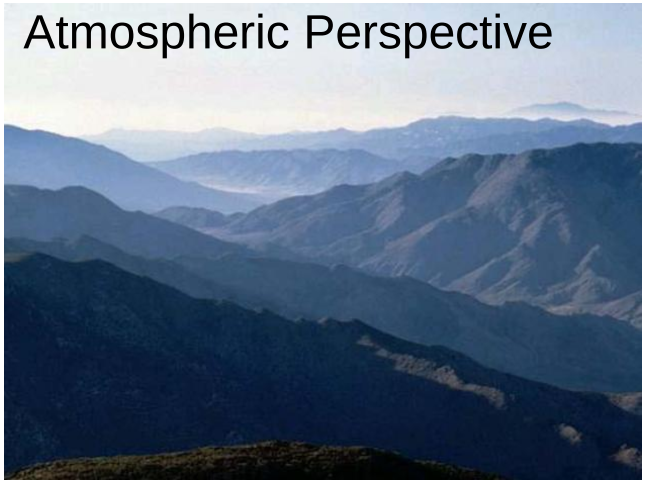
From *The Art of Photography*, Canon

Visual cues

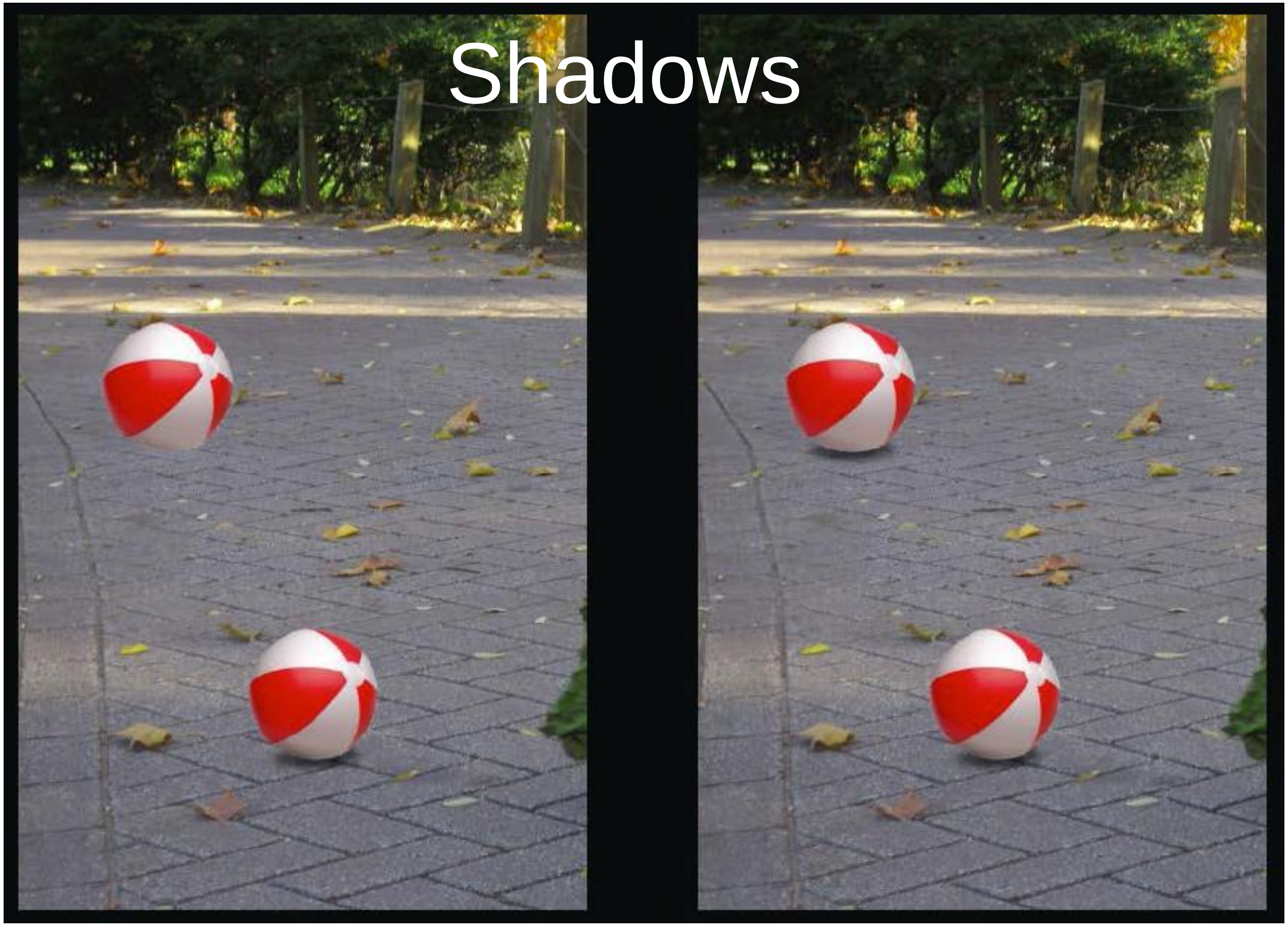
- Shading
- Texture
- Focus
- Motion

Visual cues

- Shading
- Texture
- Focus
- Motion
- Shape From X (X = shading, texture, focus, motion, rotation, ...)



Visual cues



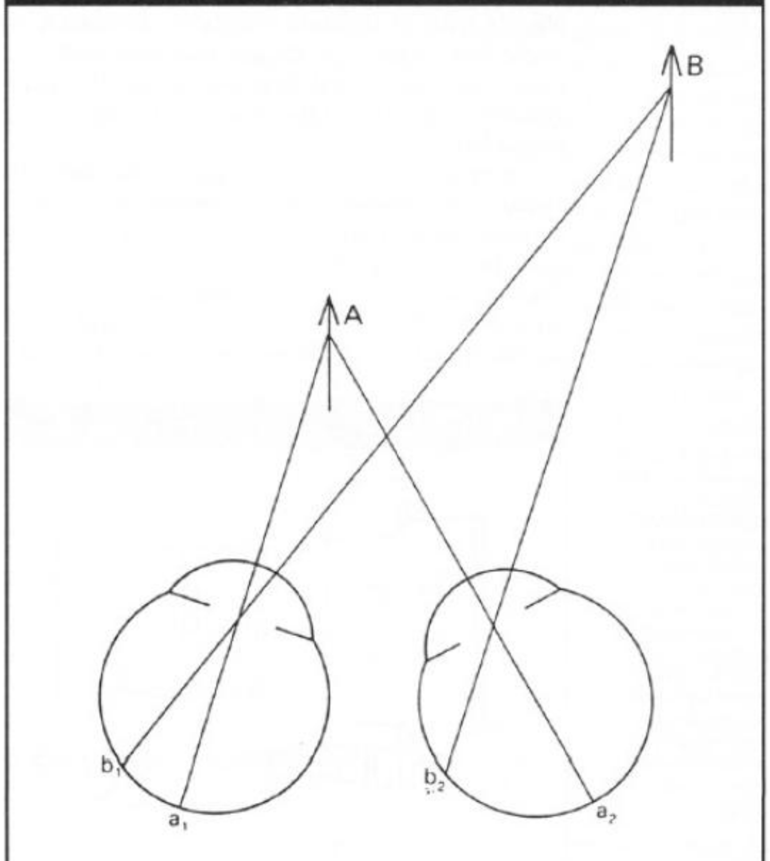
Cornell CS569 S2008, Lecture 8, slide by Steve Marschner
http://www.cs.cornell.edu/courses/cs569/2008sp/about.stm

Visual cues

- Shading
- Texture
- Focus
- Motion
- Shape From X (X = shading, texture, focus, motion, rotation, ...)
- Stereo (disparity, multi-view)

Human stereopsis: disparity

FIGURE 7.3

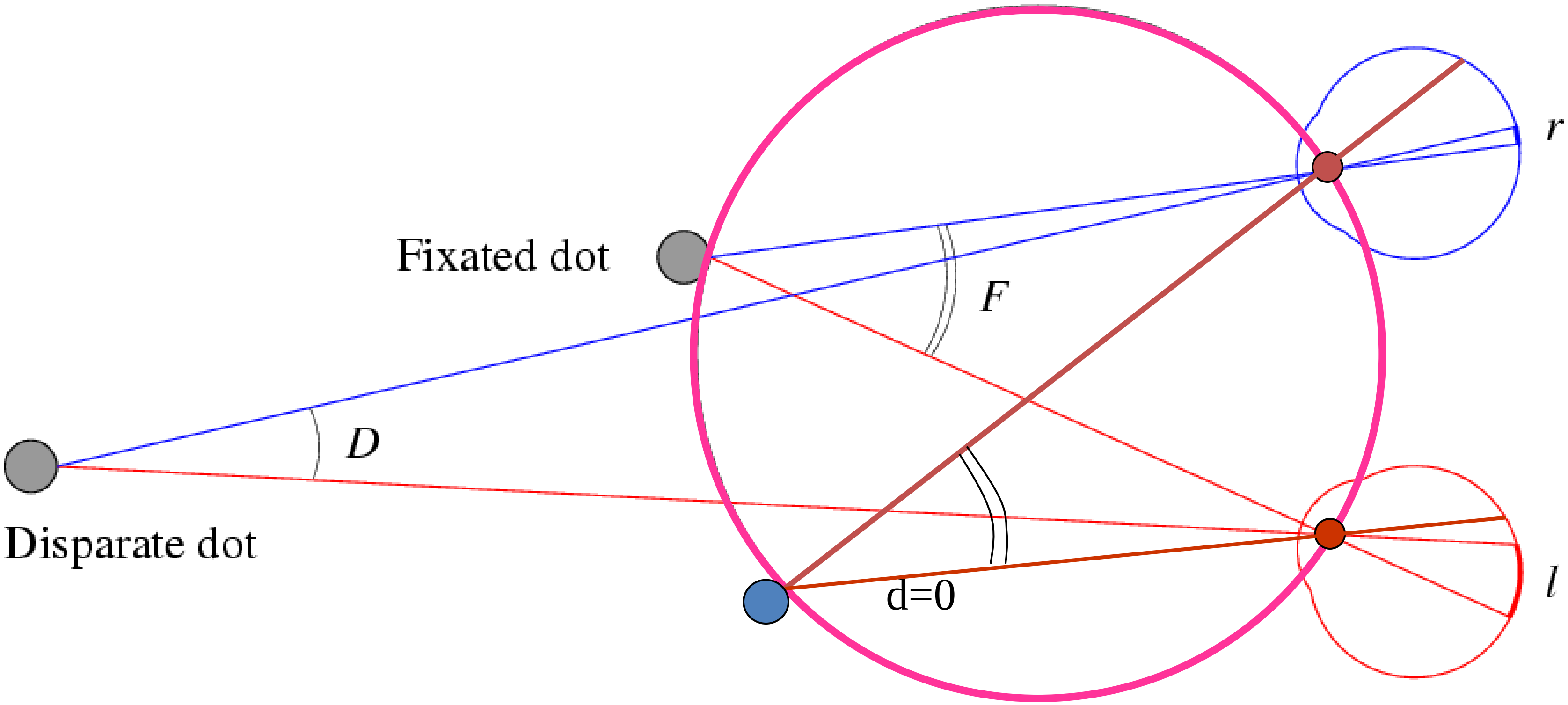


From Bruce and Green, Visual Perception, Physiology, Psychology and Ecology

Disparity occurs when eyes fixate on one object; others appear at different visual angles

Adapted from David Forsyth, UC Berkeley

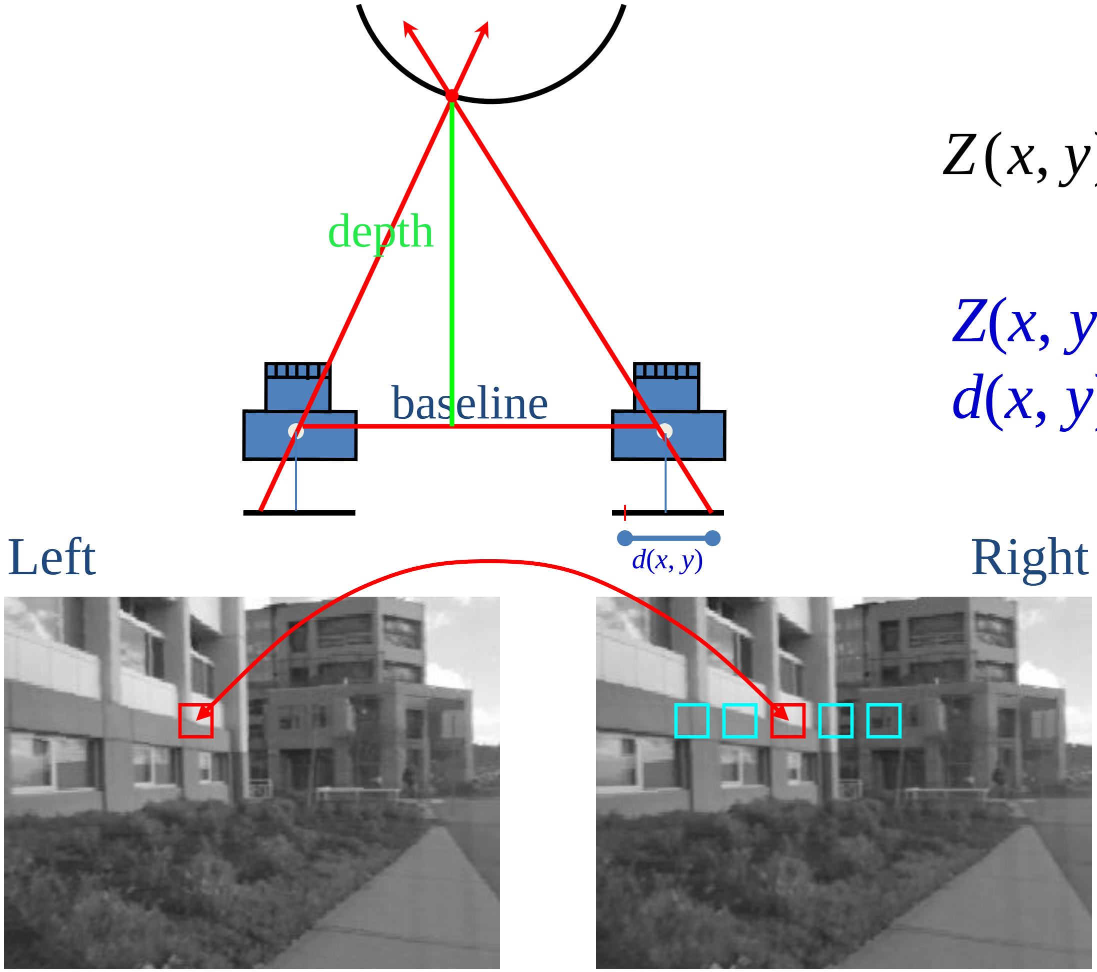
Human stereopsis: disparity



Disparity: $d = r\text{-}l = D\text{-}F.$

Adapted from M. Pollefeys

Stereo Vision



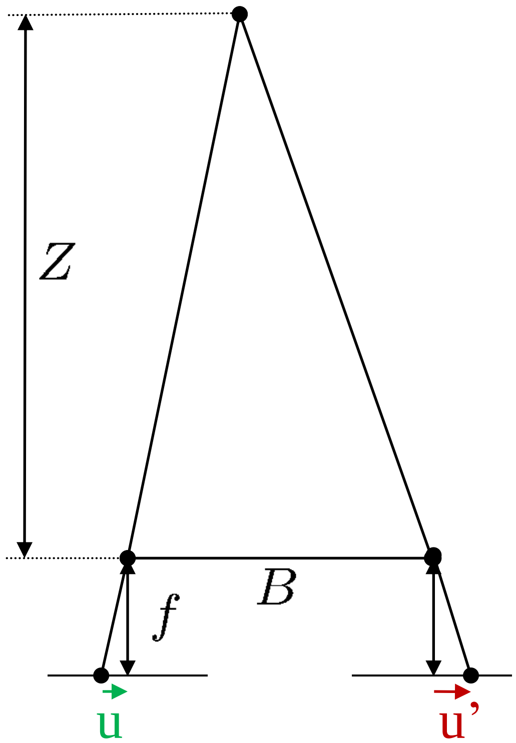
$$Z(x,y) = \frac{f\,B}{d(x,y)}$$

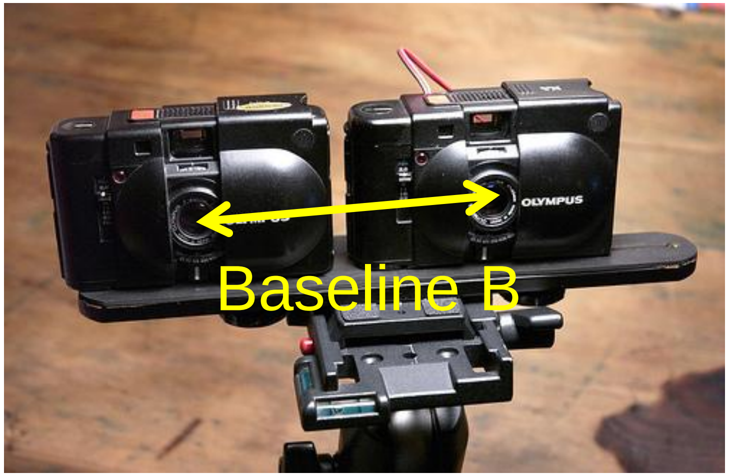
$Z(x, y)$ is depth at pixel (x, y)
$d(x, y)$ is disparity

Matching correlation windows across scan lines

J. M. Rehg © 2003

Standard stereo geometry

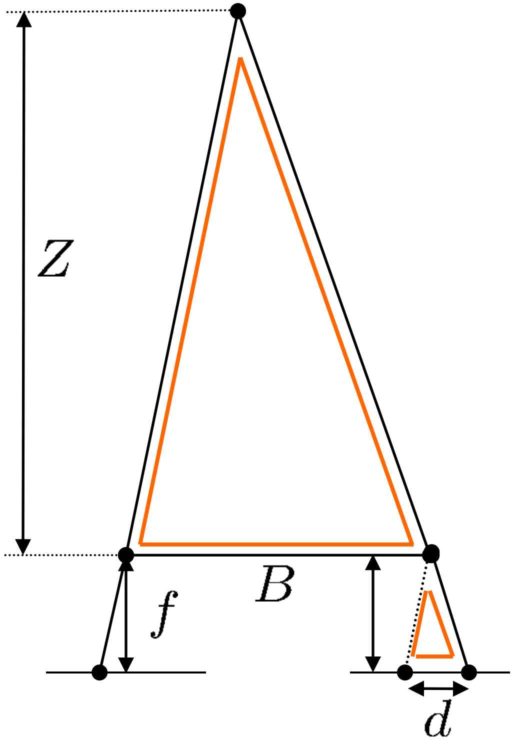




Disparity d:

$$d = |u' - u|$$

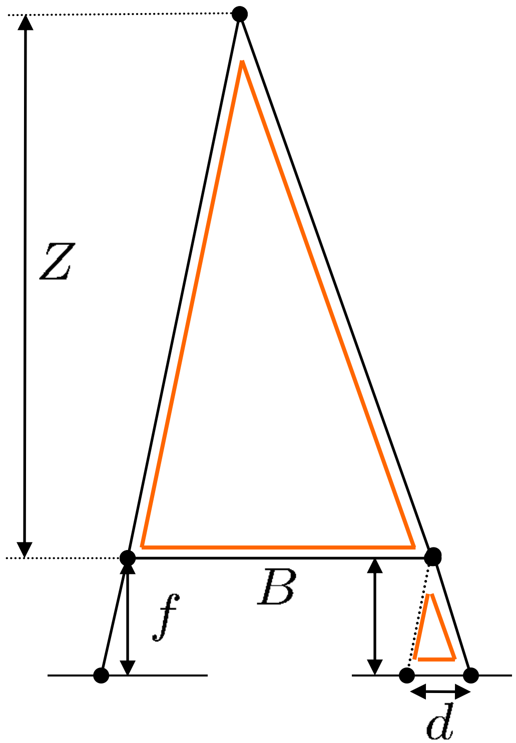
Standard stereo geometry



$$\frac{B}{Z} = \frac{d}{f}$$

$$d = \frac{Bf}{Z}$$

Standard stereo geometry

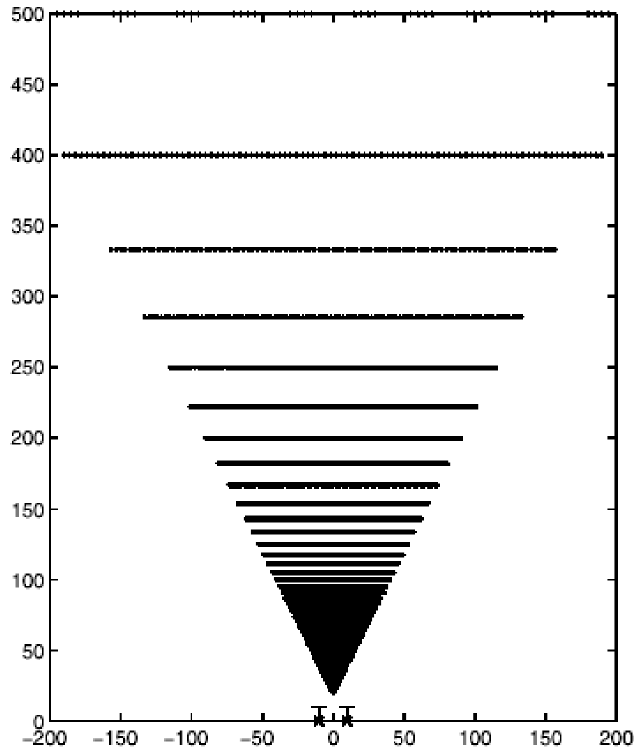


$$\frac{B}{Z} = \frac{d}{f}$$

$$d = \frac{Bf}{Z}$$

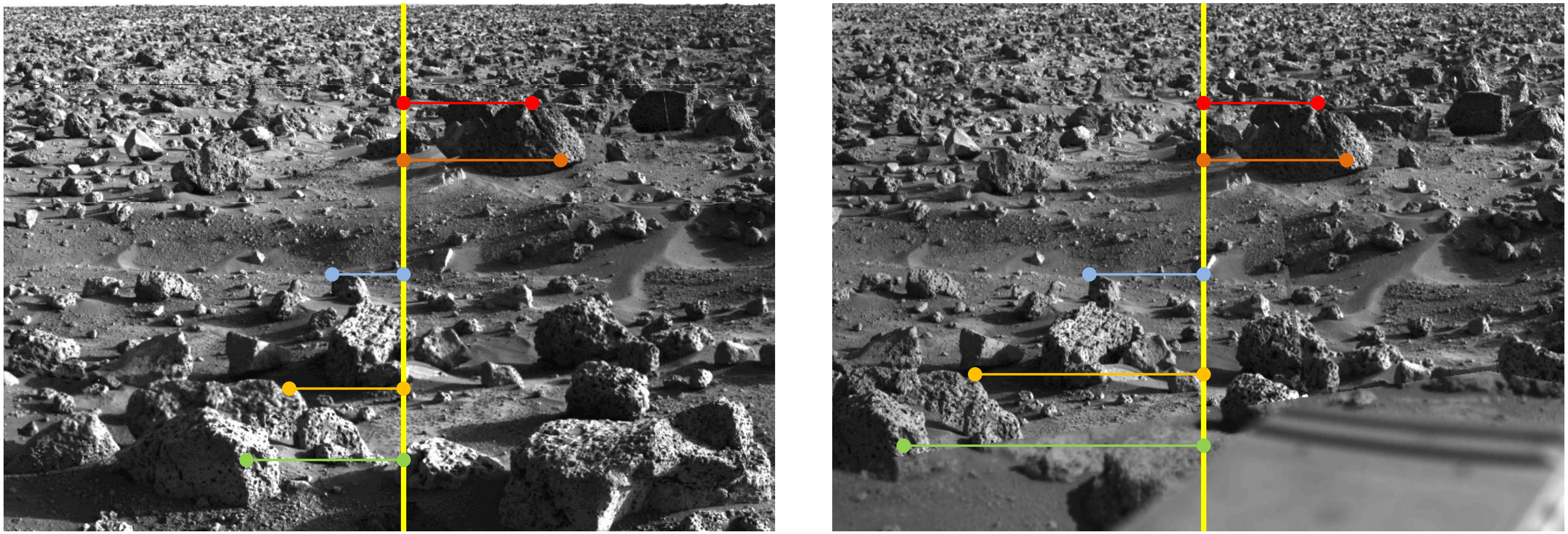
$$\frac{dd}{dZ} = \frac{Bf}{Z^2}$$

$$\Delta Z = \frac{Z^2}{Bf}\Delta d$$



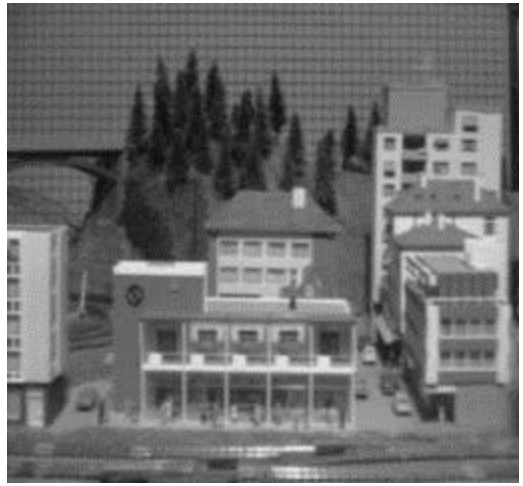
Stereo Correspondence

- Search over disparity to find correspondences
- Range of disparities to search over can change dramatically within a single image pair.

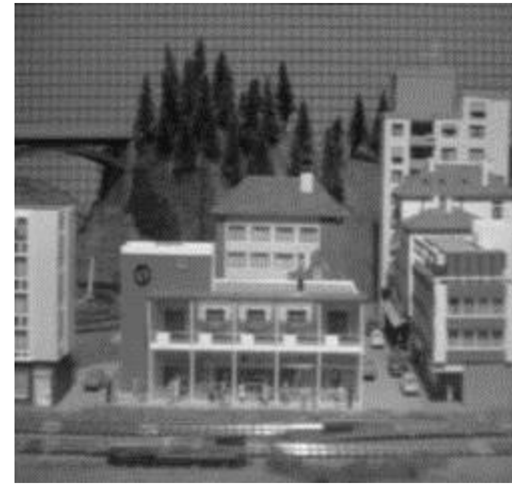


Why is disparity important?

I1

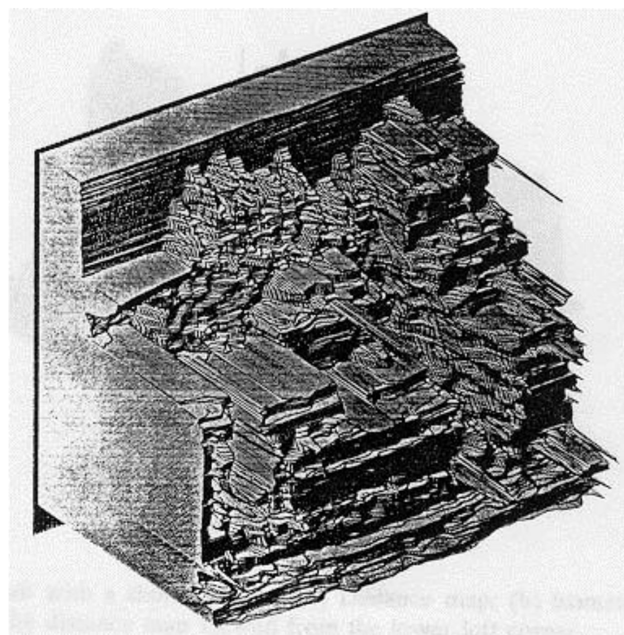


I2



I10

Given dense disparity map, we can calculate a depth/distance/ range map.



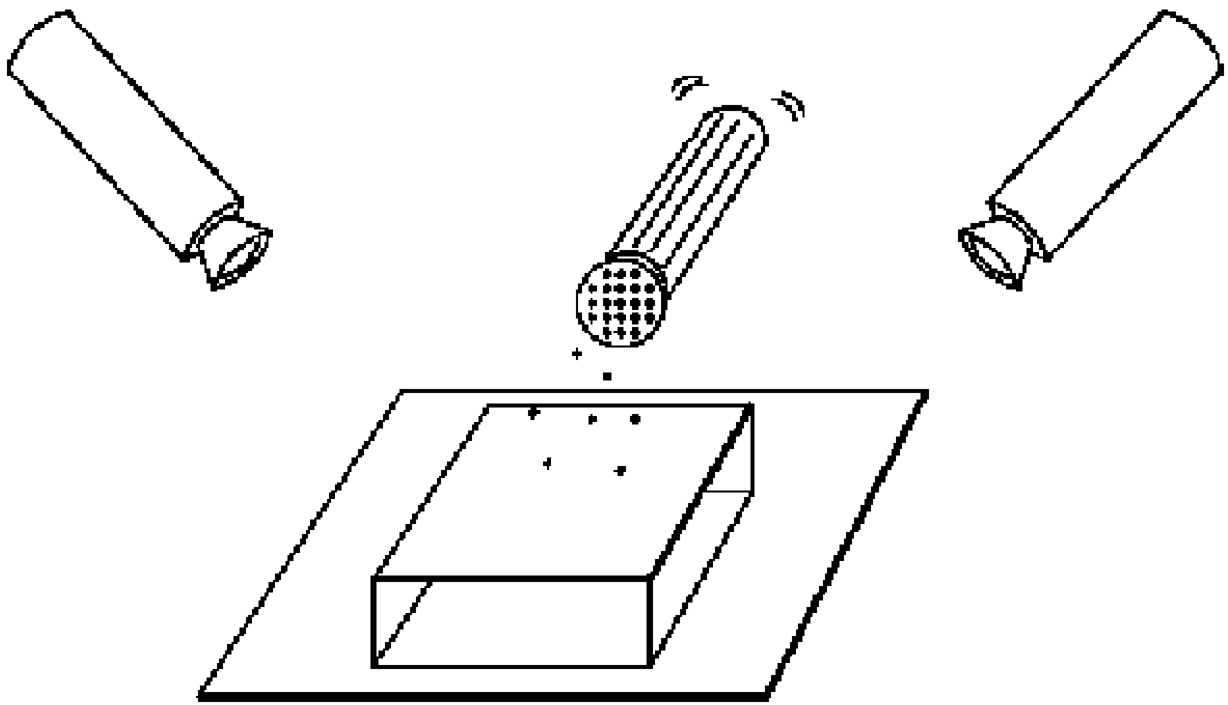
Reprinted from “A Multiple-Baseline Stereo System,” by M. Okutami and T. Kanade, IEEE Trans. on Pattern Analysis and Machine Intelligence, 15(4):353-363 (1993). \copyright 1993 IEEE.

Random dot stereograms

- Julesz 1960: Do we identify local brightness patterns before fusion (monocular process) or after (binocular)?

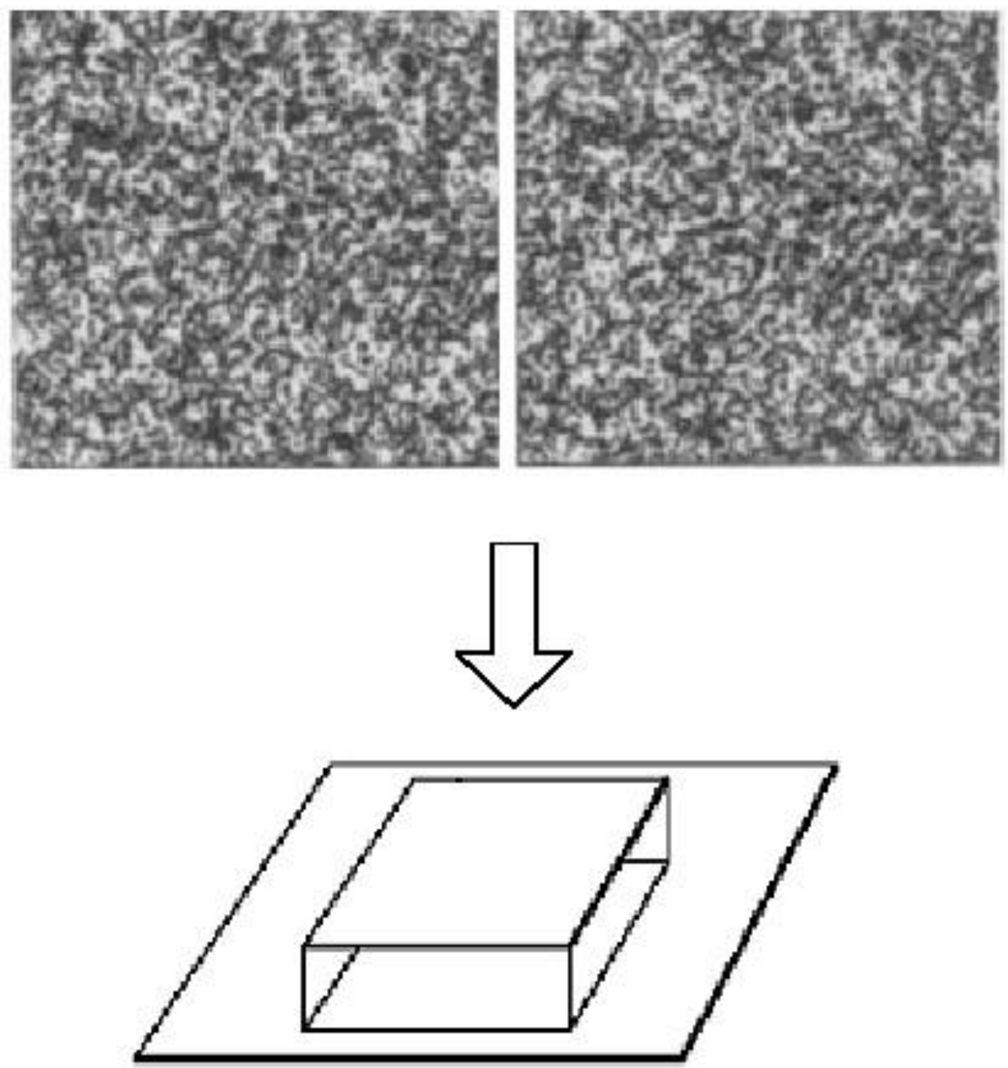
- To test: pair of synthetic images obtained by randomly spraying black dots on white objects

Random dot stereograms

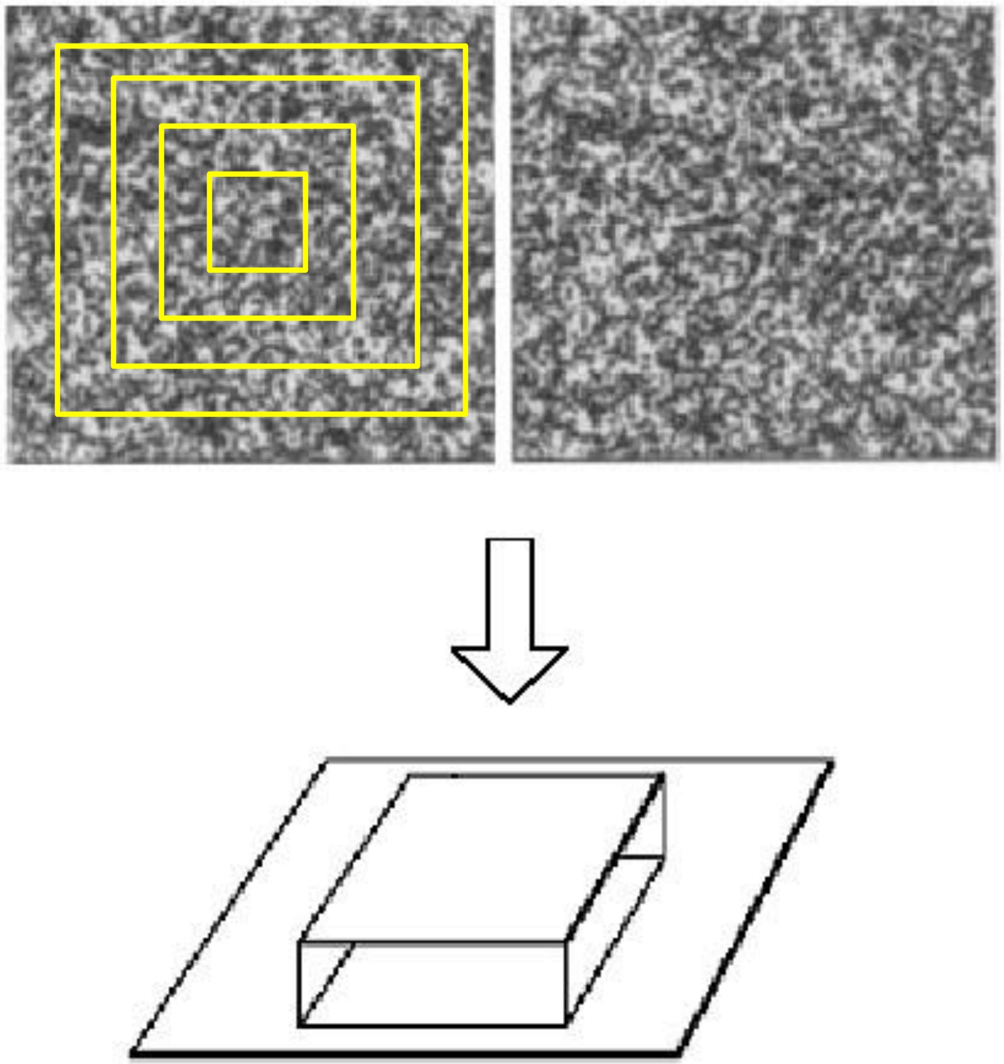


Forsyth & Ponce

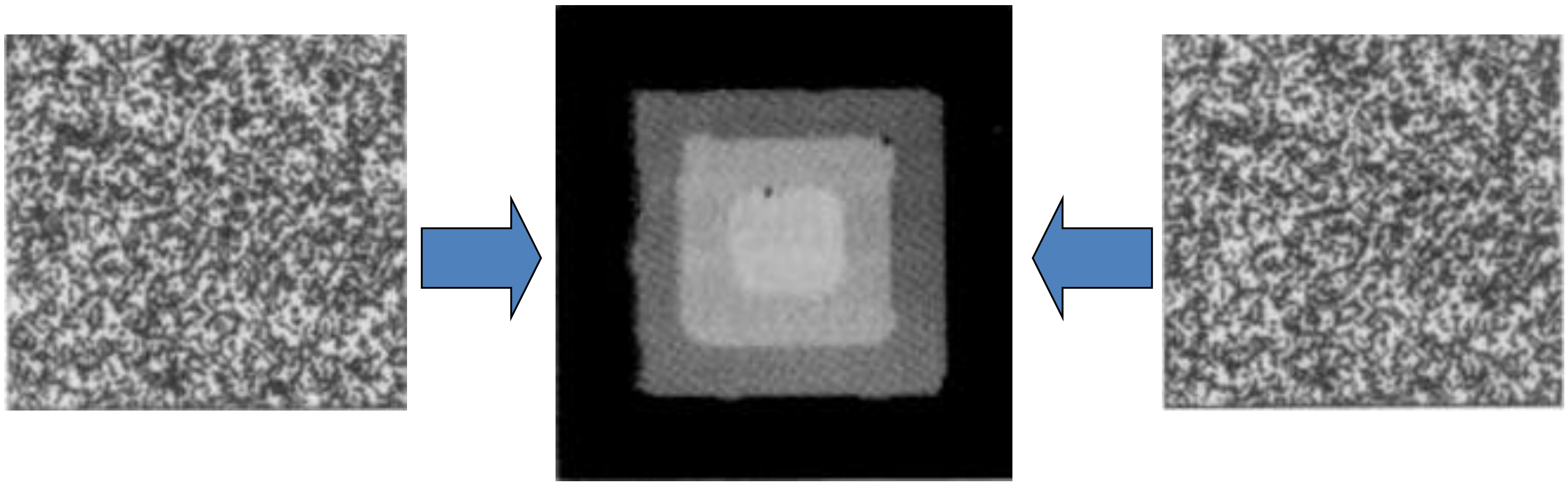
Random dot stereograms



Random dot stereograms



A Cooperative Model (Marr and Poggio, 1976



Random dot stereograms

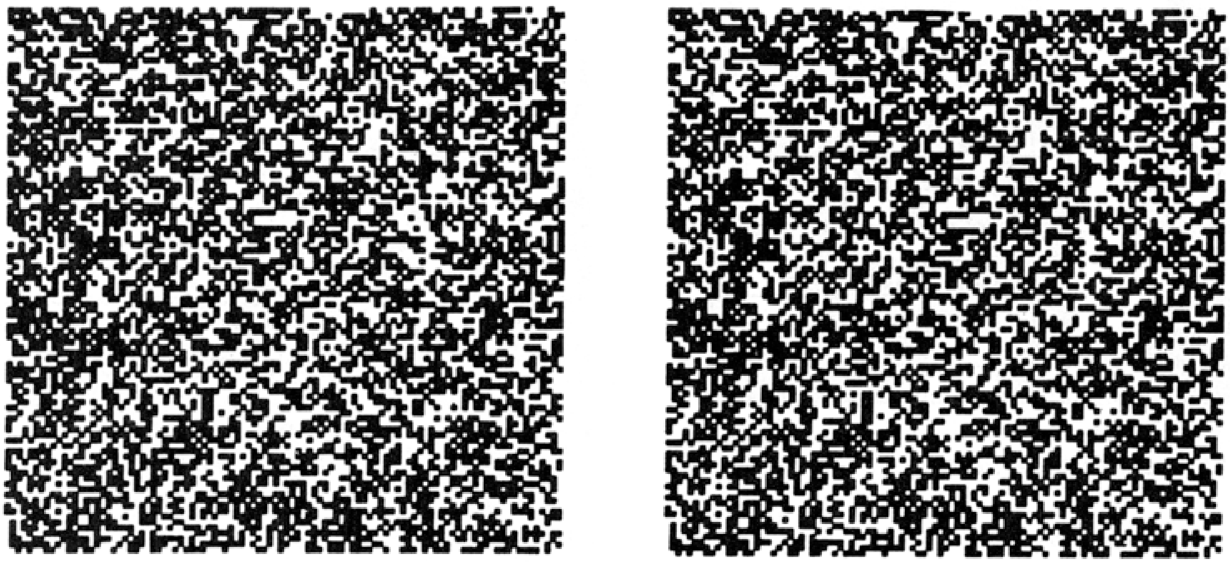


Figure 5.3.8 A random dot stereogram. These two images are derived from a single array of randomly placed squares by laterally displacing a region of them as described in the text. When they are viewed with crossed disparity (by crossing the eyes) so that the right eye's view of the left image is combined with the left eye's view of the right image, a square will be perceived to float above the page. (See pages 210–211 for instructions on fusing stereograms.)

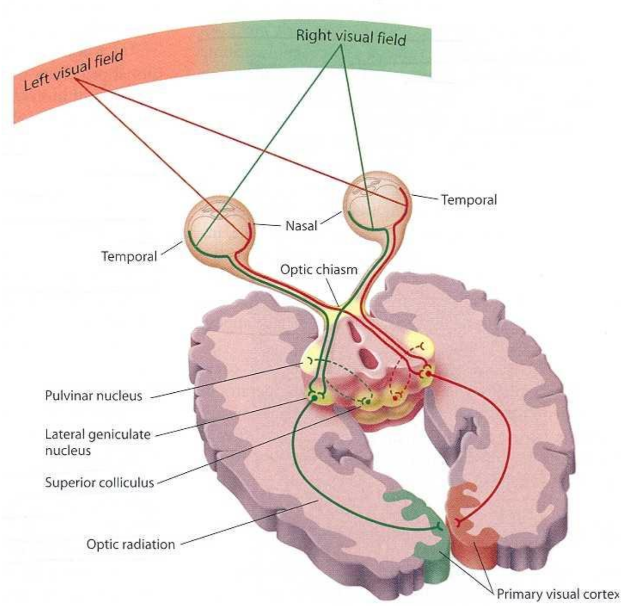
From Palmer, "Vision Science", MIT Press

Random dot stereograms

- When viewed monocularly, they appear random; when viewed stereoscopically, see 3d structure.

- Conclusion: human binocular fusion not directly associated with the physical retinas; must involve the central nervous system

- Imaginary* *“cyclopean retina”* that combines the left and right image stimuli as a single unit



[Visual Pathway.jpg](#) wiki.ucl.ac.uk

*This was because it was as though we have a cyclopean eye inside our brains that can see cyclopean stimuli hidden to each of our actual eyes.

Grauman

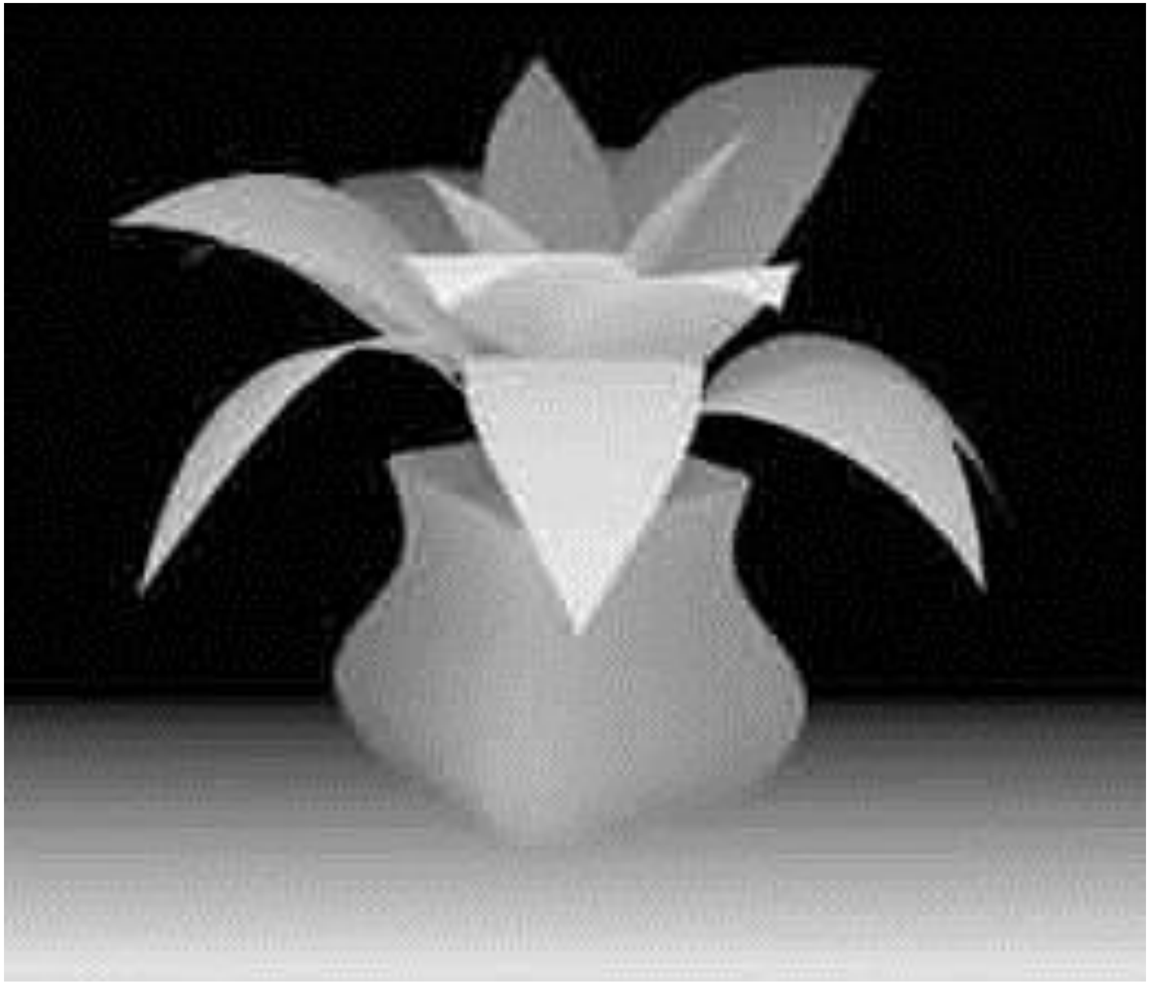
Autostereograms

Exploit disparity as depth cue using single image

(Single image random dot stereogram, Single image stereogram)

Images from magiceye.com

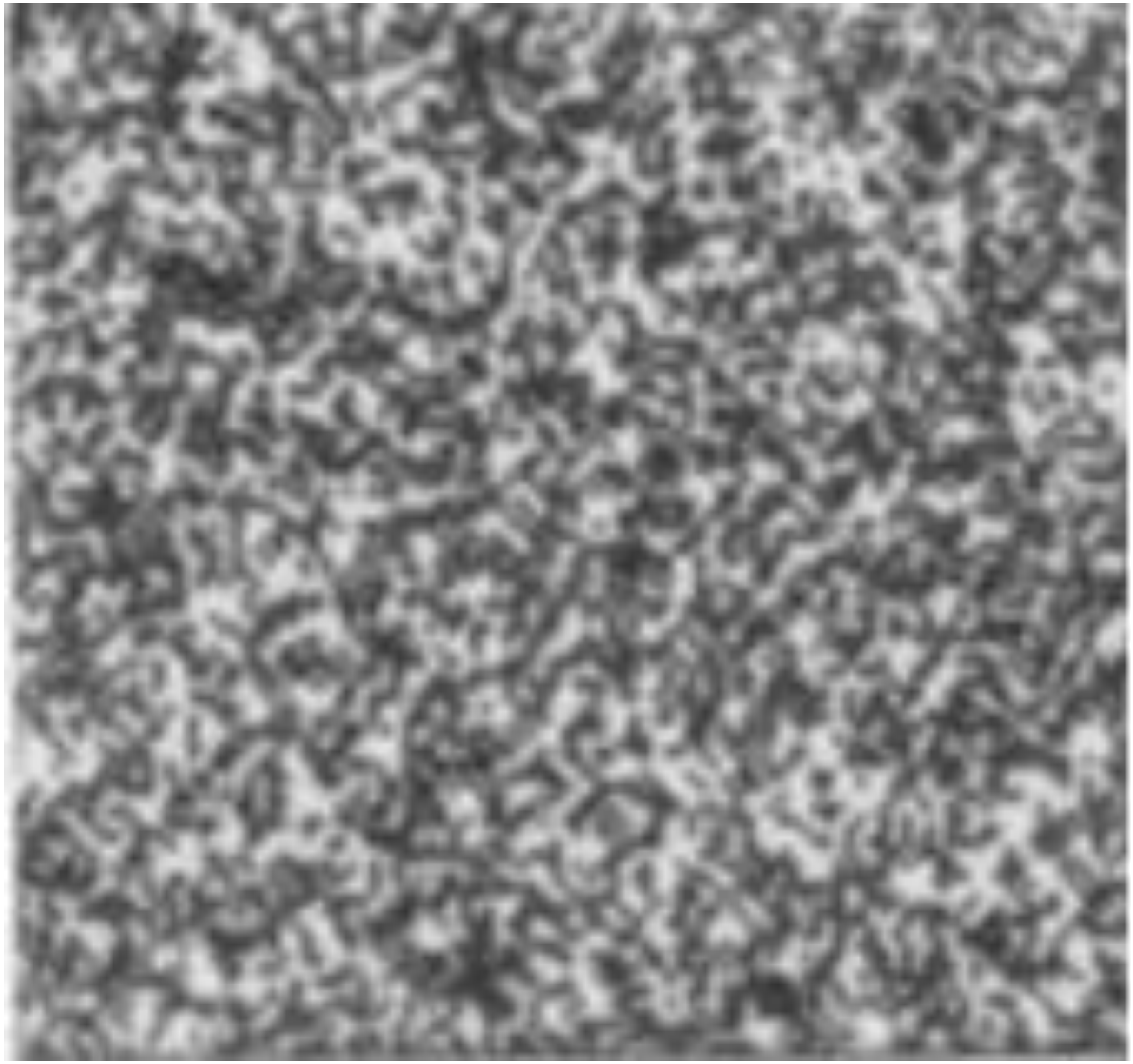
Autostereograms



Images from magiceye.com

Optical flow

Where do pixels move?



Optical flow

Where do pixels move?

http://www.well.com/~jimg/stereo/stereo_list.html

Grauman

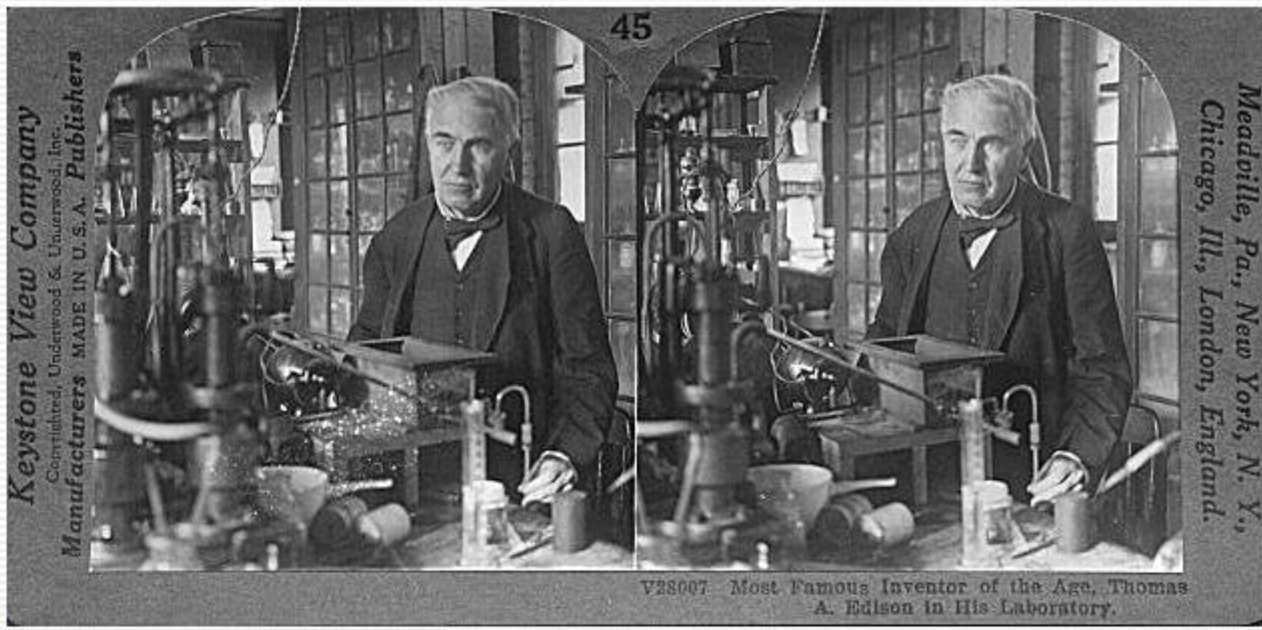
Stereo photography and stereo viewers

Take two pictures of the same subject from two slightly different viewpoints and display so that each eye sees only one of the images.

Invented by Sir Charles Wheatstone, 1838

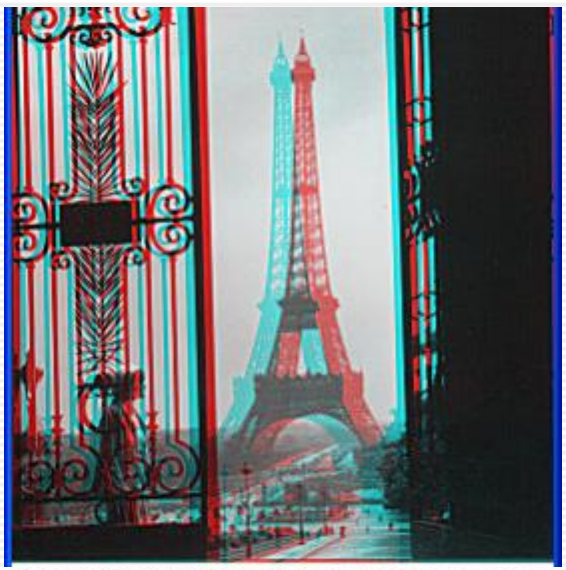
Image courtesy of fisher-price.com

Grauman



http://www.johnsonshawmuseum.org

Grauman



http://www.johnsonshawmuseum.org

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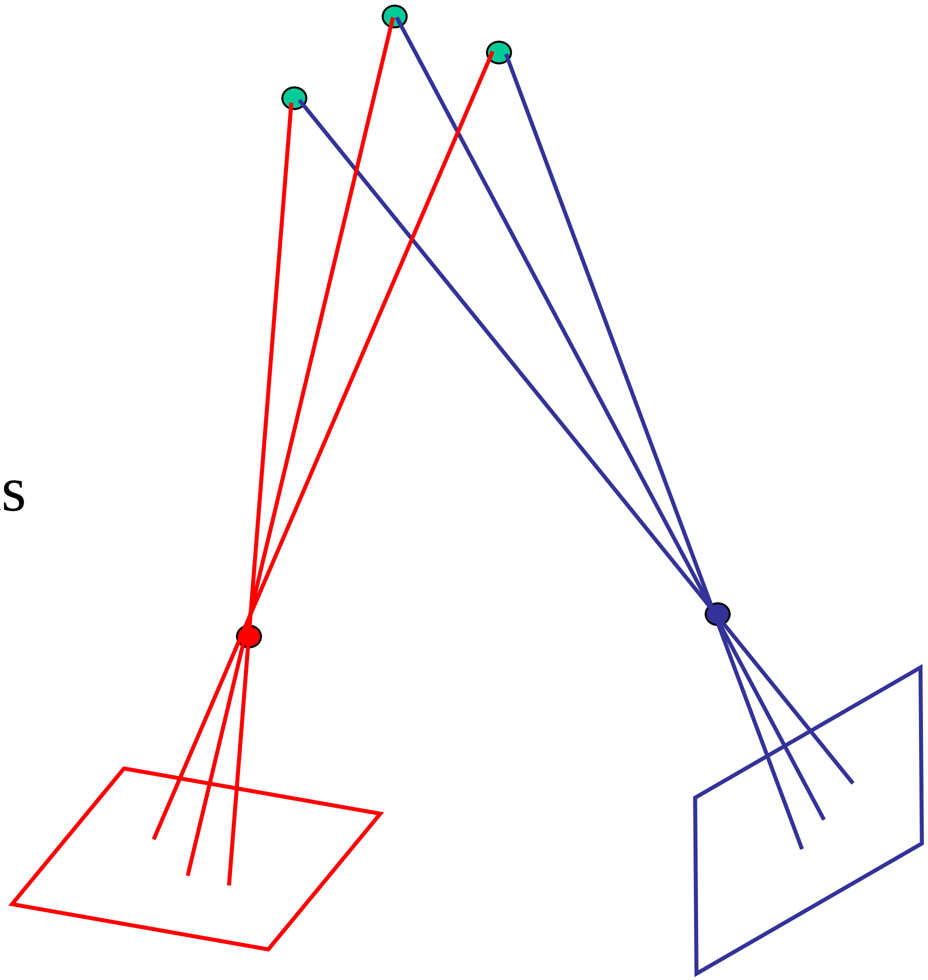
Public Library, Stereoscopic Looking Room, Chicago, by Phillips, 1923

Grauman

Multi-View Geometry

Relates

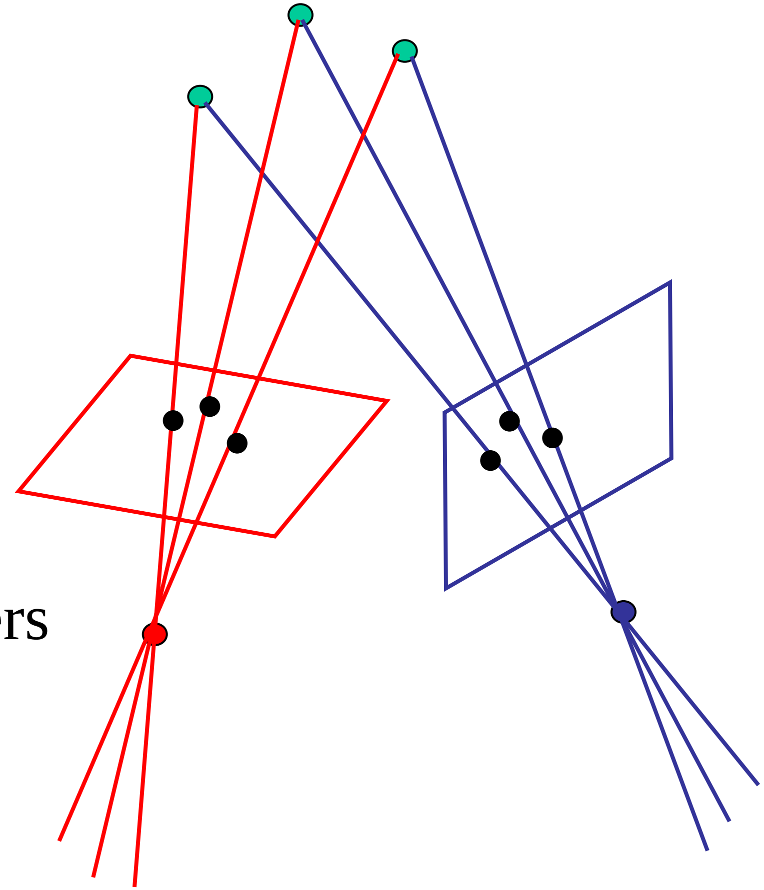
- 3D World Points
- Camera Centers
- Camera Orientations



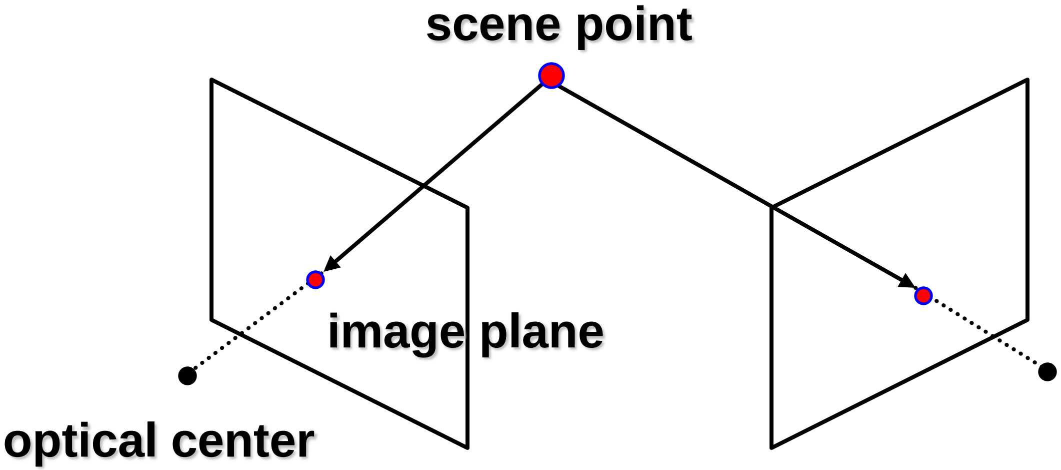
Multi-View Geometry

Relates

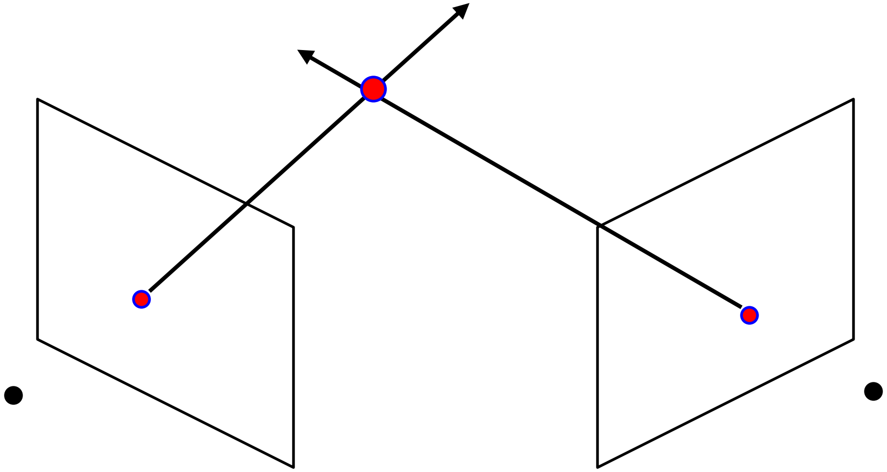
- 3D World Points
- Camera Centers
- Camera Orientations
- Camera Intrinsic Parameters
- Image Points



Stereo



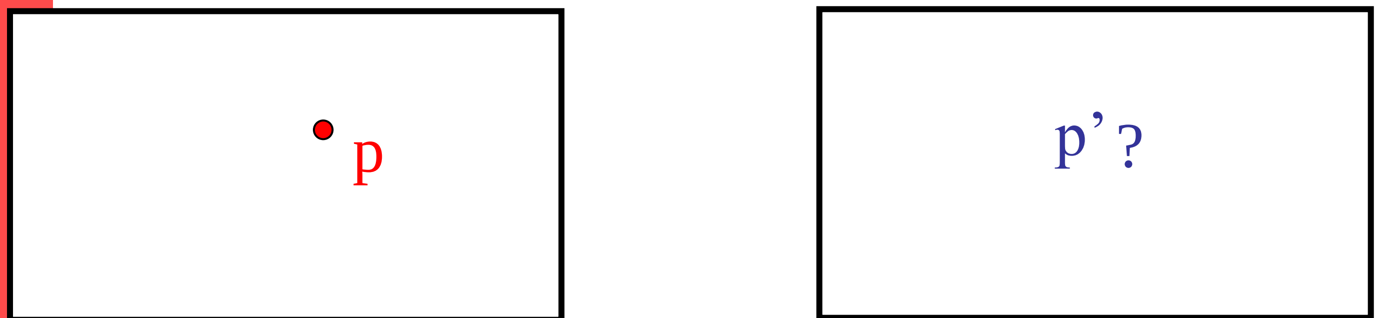
Stereo



Basic Principle: Triangulation

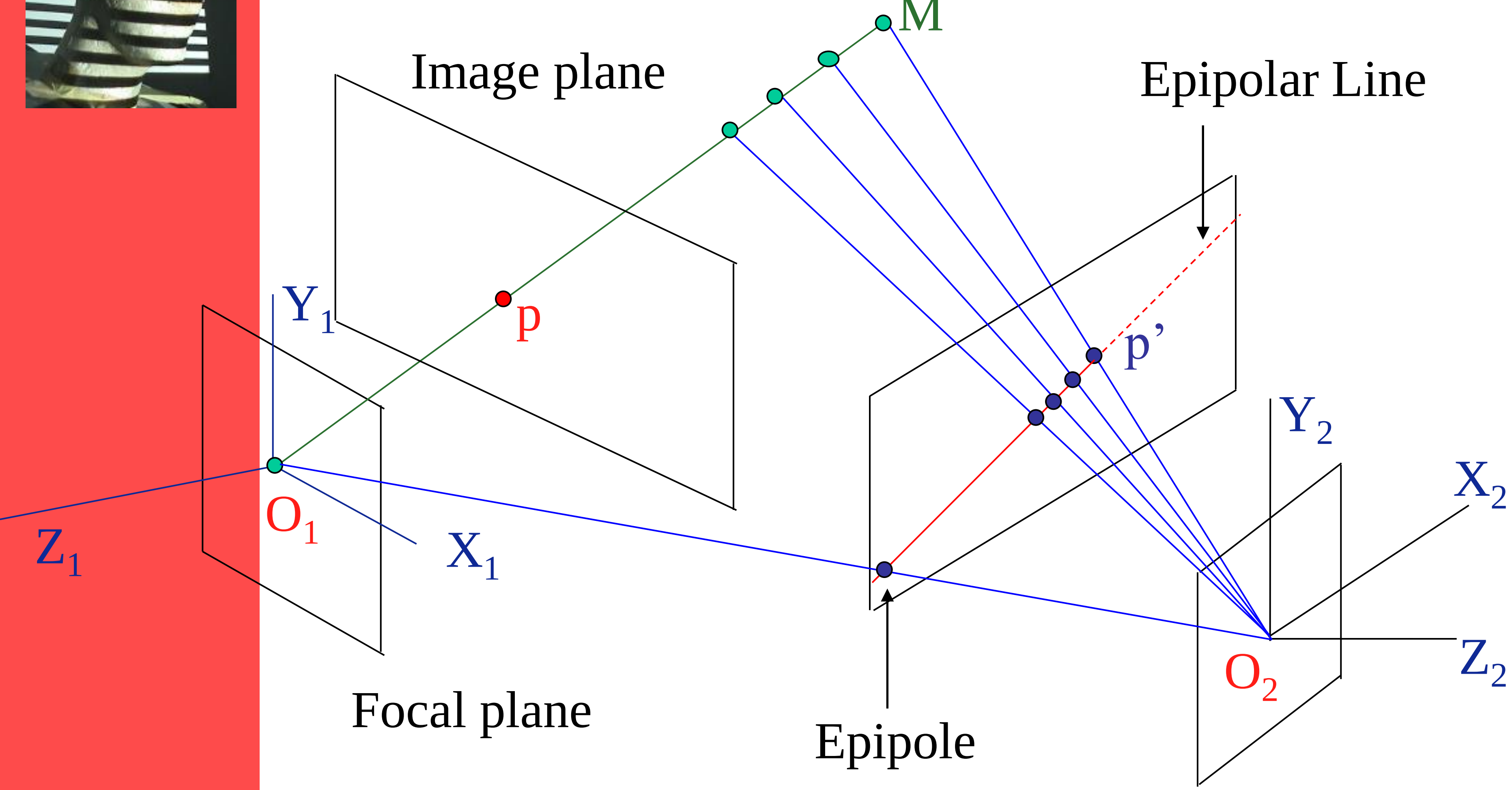
- Gives reconstruction as intersection of two rays
- Requires
  - calibration
  - ***point correspondence***

Stereo Constraints



Given p in left image, where can the corresponding point p’ in right image be?

Stereo Constraints

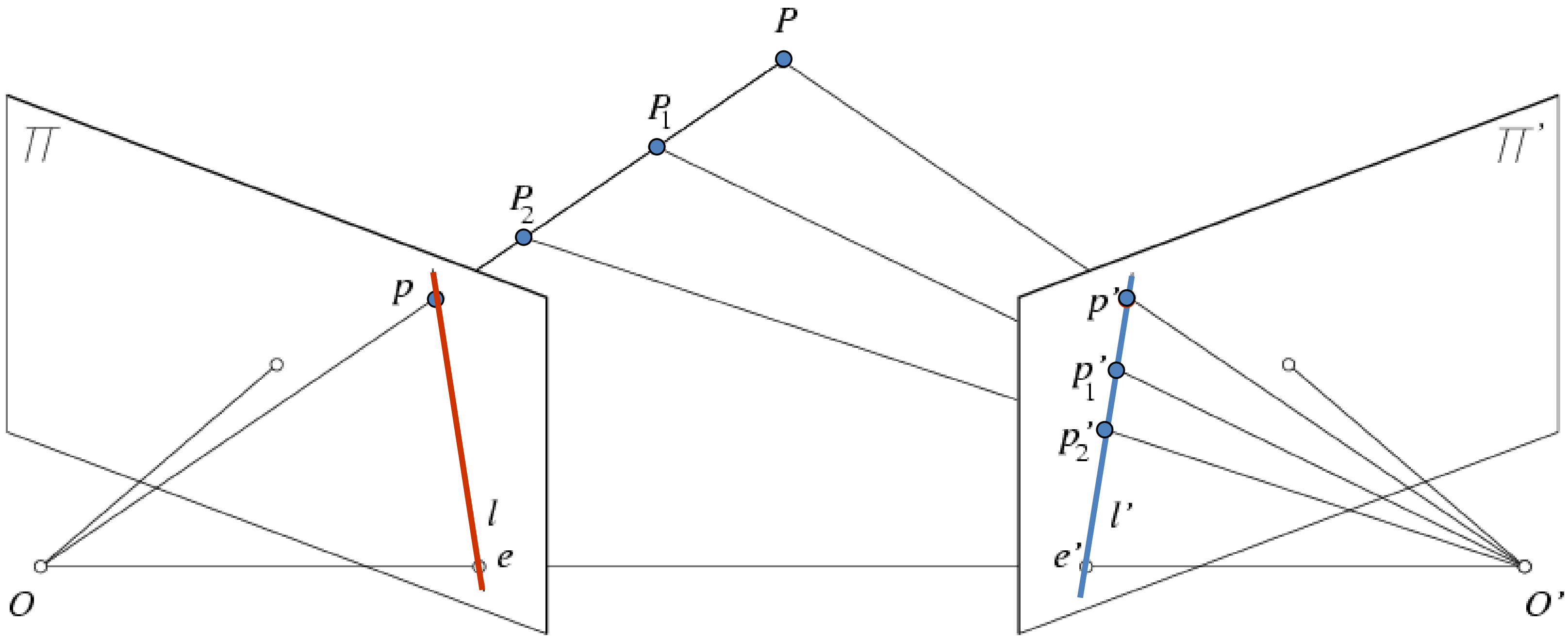


Demo Epipolar Geometry

Java Applet

credit to:
Quang-Tuan Luong
SRI Int.
Sylvain Bougnoux

Epipolar constraint



- Potential matches for *p* have to lie on the corresponding epipolar line *l'*.
- Potential matches for *p'* have to lie on the corresponding epipolar line *l*.

http://www.ai.sri.com/~luong/research/Meta3DViewer/EpipolarGeo.html

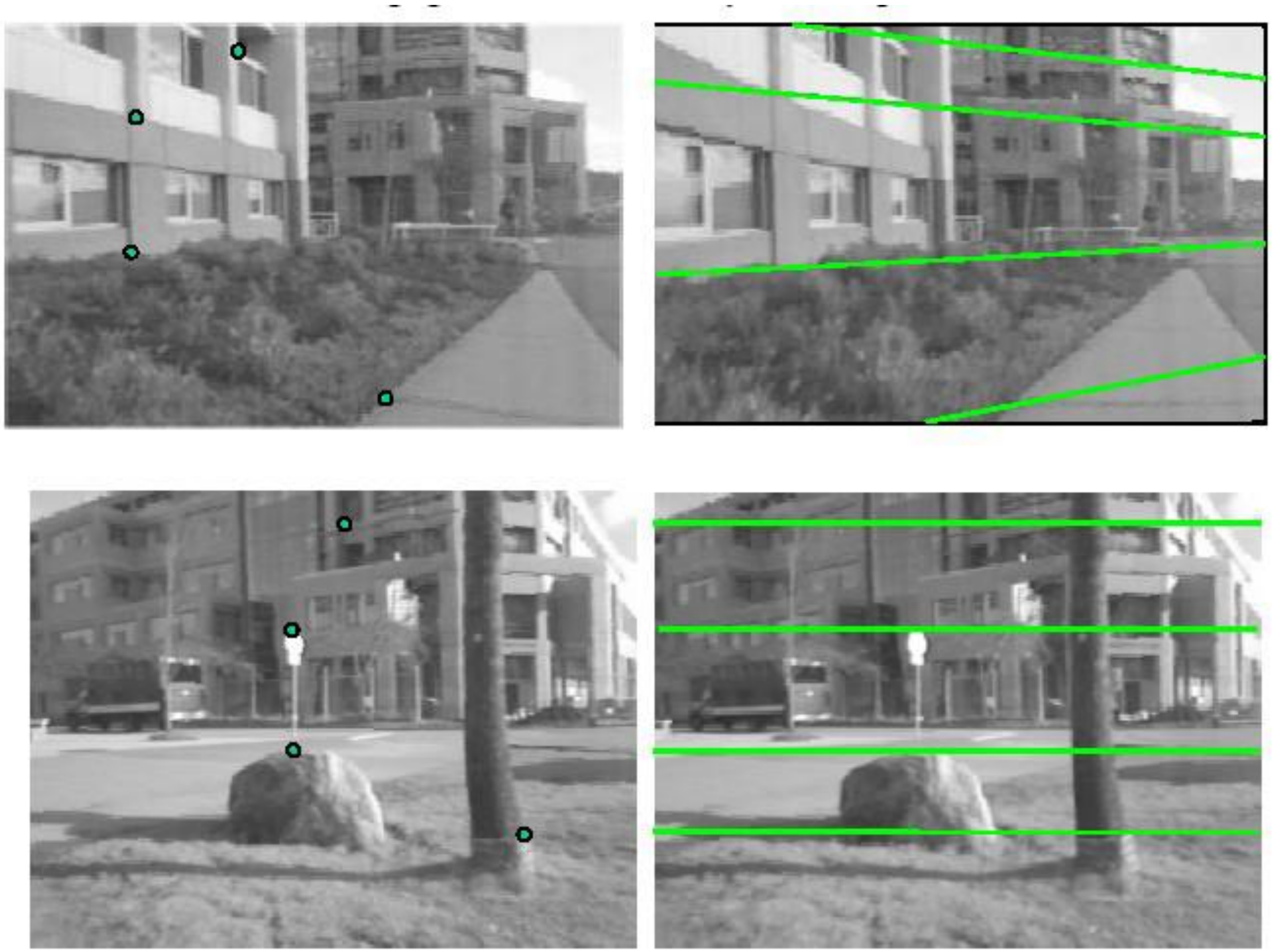
Source: M. Pollefeys

Finding Correspondences

Andrea Fusiello, CVonline

Strong constraints for searching for corresponding points!

Example



Parallel Cameras: Corresponding points on horizontal lines.

Epipolar Constraint

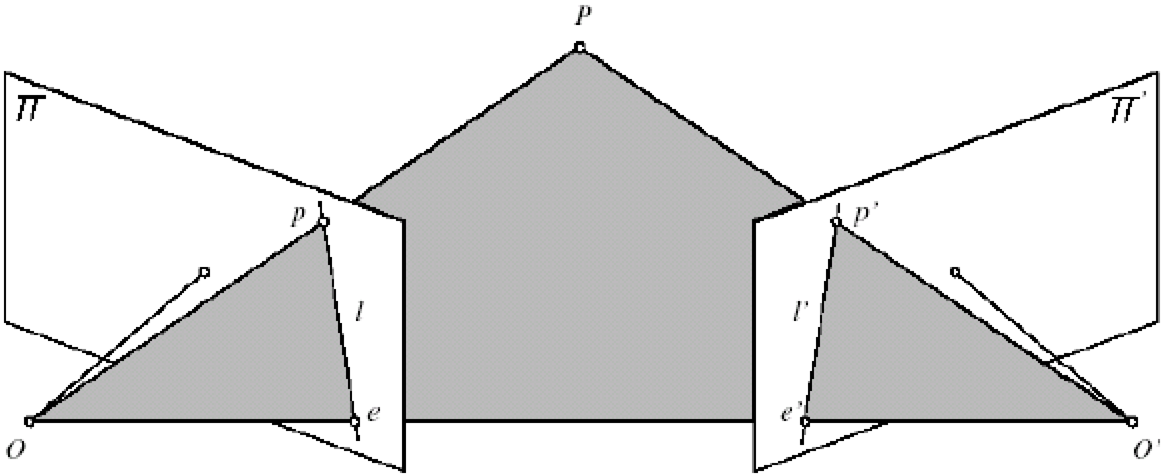


FIGURE 11.1: Epipolar geometry: the point P, the optical centers O and O' of the two cameras, and the two images p and p' of P all lie in the same plane.

All epipolar lines contain epipole, the image of other camera center.

From Geometry to Algebra

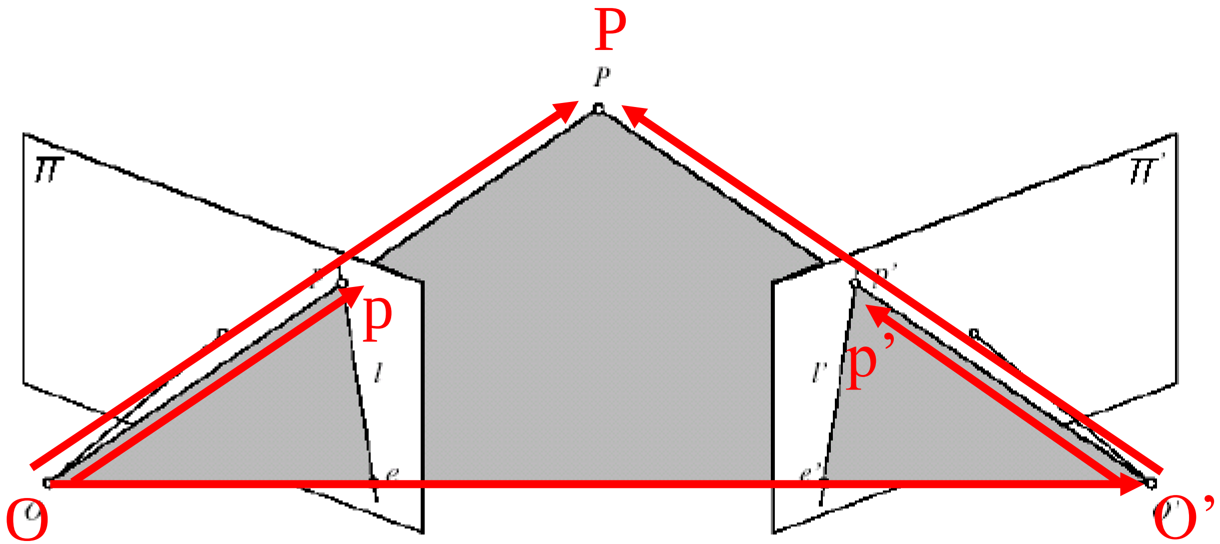
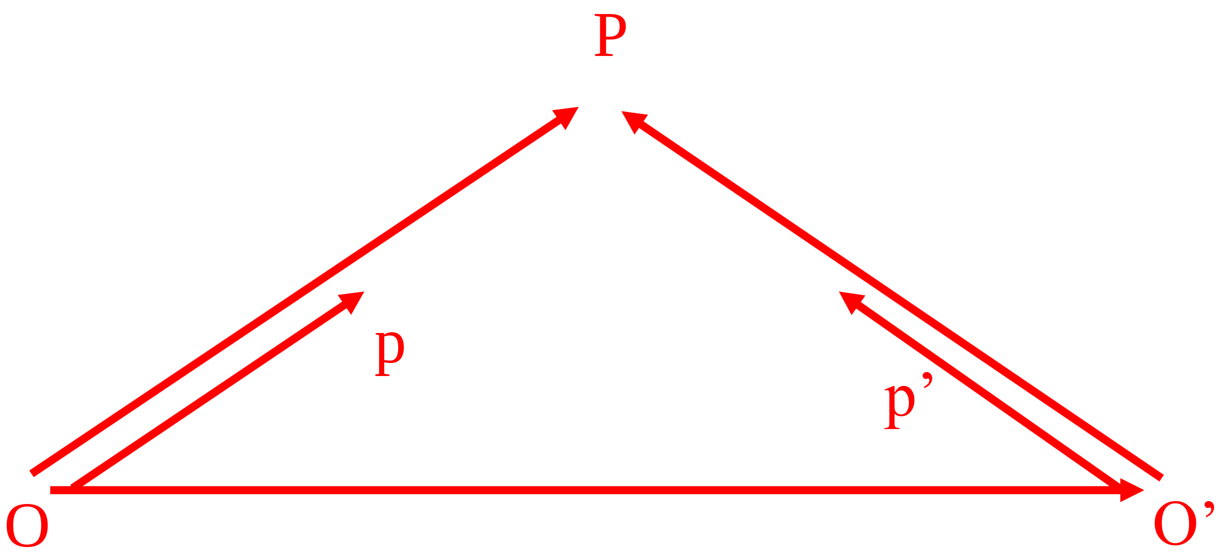


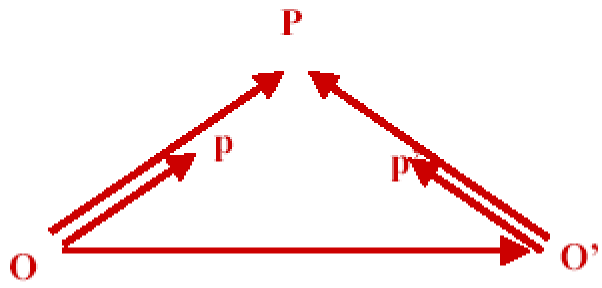
FIGURE 11.1: Epipolar geometry: the point P, the optical centers O and O' of the two cameras, and the two images p and p' of P all lie in the same plane.

From Geometry to Algebra

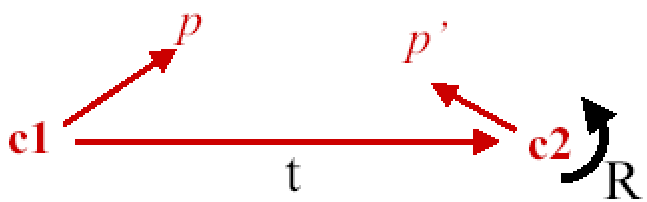


The epipolar constraint: these vectors are coplanar:

$$\overrightarrow{Op} \cdot [\overrightarrow{OO'} \times \overrightarrow{O'p'}] = 0$$



$$\overrightarrow{Op} \cdot [\overrightarrow{OO'} \times \overrightarrow{O'p'}] = 0$$



p,p' are image coordinates of P in c1 and c2…

c2 is related to c1 by rotation R and translation t

$$\boldsymbol{p} \cdot [\boldsymbol{t} \times (\mathcal{R}\boldsymbol{p}')] = 0$$

Linear Constraint:
Should be able to express as matrix multiplication.

Review: Matrix Form of Cross Product

The vector cross product also acts on two vectors and returns a third vector. Geometrically, this new vector is constructed such that its projection onto either of the two input vectors is zero.

$$\vec{a} \times \vec{b} = \begin{bmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{bmatrix}$$

$$\vec{a} \times \vec{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = \vec{c}$$

$$\vec{a} \cdot \vec{c} = 0$$

$$\vec{b} \cdot \vec{c} = 0$$

Review: Matrix Form of Cross Product

$$\vec{a} \times \vec{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = \vec{c}$$

$$\vec{a} \cdot \vec{c} = 0$$

$$\vec{b} \cdot \vec{c} = 0$$

$$[a_x] = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

$$\boxed{\vec{a} \times \vec{b} = [a_x]\vec{b}}$$

Matrix Form

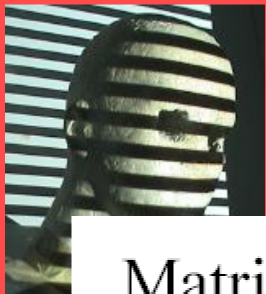
$$\boldsymbol{p} \cdot [\boldsymbol{t} \times (\mathcal{R}\boldsymbol{p}')] = 0$$

$$\vec{a} \times \vec{b} = [a_x]\vec{b}$$

$$p^T [t_x] \Re p' = 0$$

$$\varepsilon = [t_x] \Re$$

$$\boldsymbol{p}^T \mathcal{E} \boldsymbol{p}' = 0$$

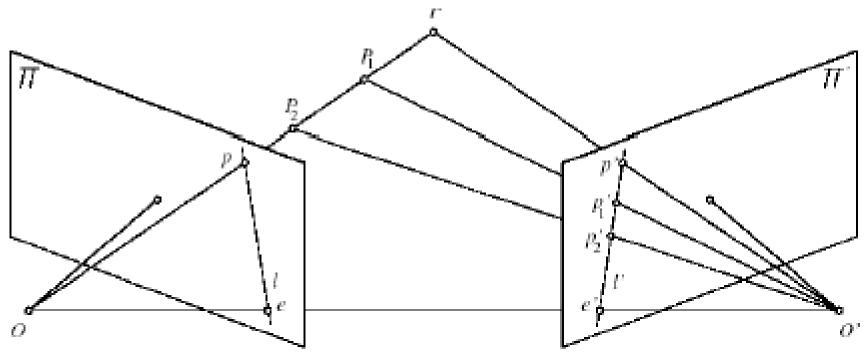


The Essential Matrix

Matrix that relates image of point in one camera to a second camera, given translation and rotation.

$$\mathcal{E} = [t_x]\Re$$

$$\boldsymbol{p}^T \mathcal{E} \boldsymbol{p}' = 0$$



$$\vec{a} \times \vec{b} = [a_x]\vec{b}$$

The Essential Matrix

- Based on the Relative Geometry of the Cameras
- Assumes Cameras are calibrated (i.e., intrinsic parameters are known)
- Relates image of point in one camera to a second camera (points in camera coordinate system).
- Is defined up to scale
- 5 independent parameters

The Essential Matrix

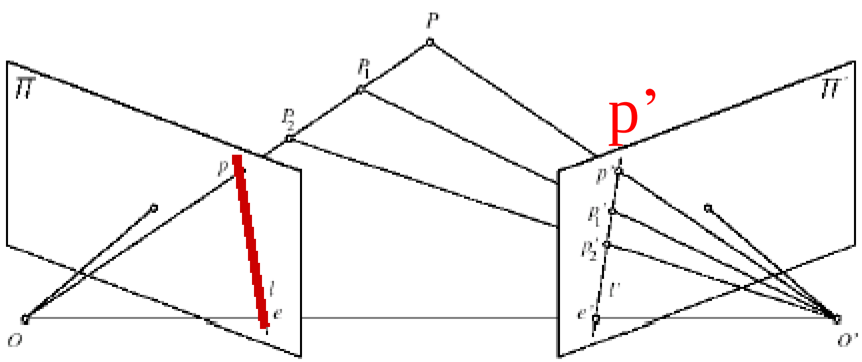
$$\boldsymbol{p}^T \mathcal{E} \boldsymbol{p}' = 0$$

What is $\mathcal{E}$p' ?

The Essential Matrix

$\mathcal{E}p'$ is the epipolar line corresponding to p' in the left camera.

$$au + bv + c = 0$$



$\mathcal{E}p'$

$$p = (u, v, 1)^T$$

$$l = (a, b, c)^T$$

$$l \cdot p = 0$$

$$\mathcal{E}p' \cdot p = 0$$

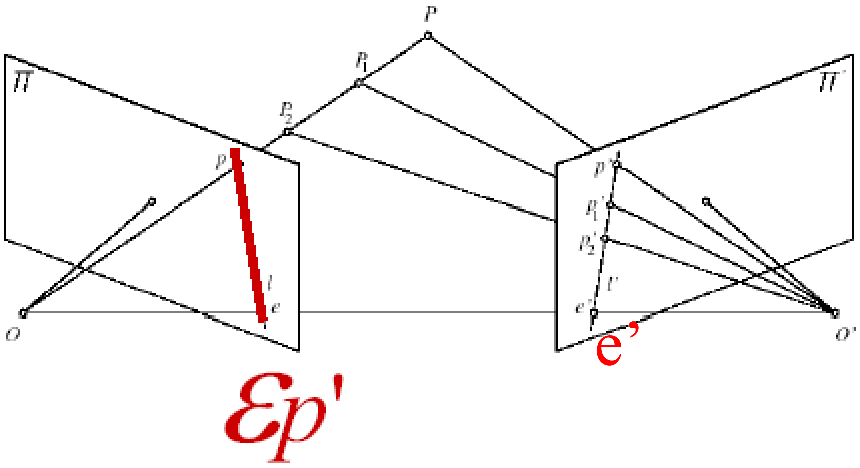
$$\mathbf{p}^T \mathcal{E} \mathbf{p}' = 0$$

Similarly $\mathcal{E}^T p$ is the epipolar line corresponding to **p** in the right camera

The Essential Matrix

$e^T \mathcal{E} e'$?

What is $\mathcal{E} e'$?

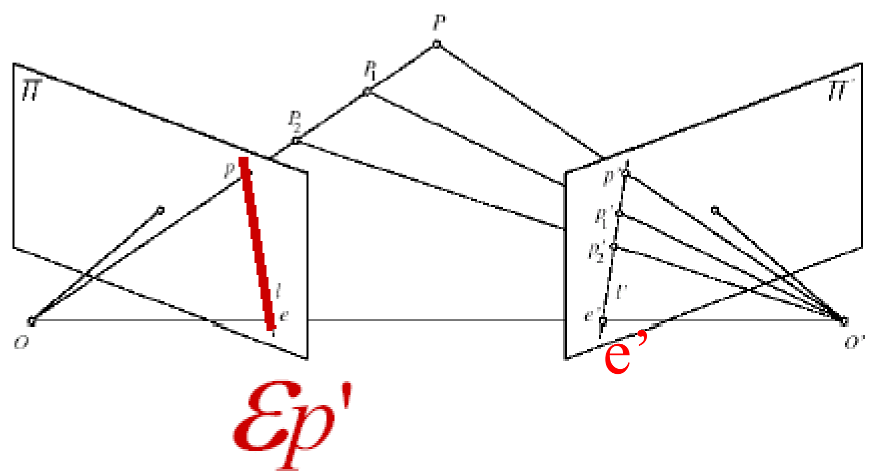


The Essential Matrix

$$\mathcal{E}e' = [t_\times]Re' = 0$$

Similarly, $\mathcal{E}^T e = R^T[t_\times]^T e = -R^T[t_\times]e = 0$

Essential Matrix is singular with rank 2

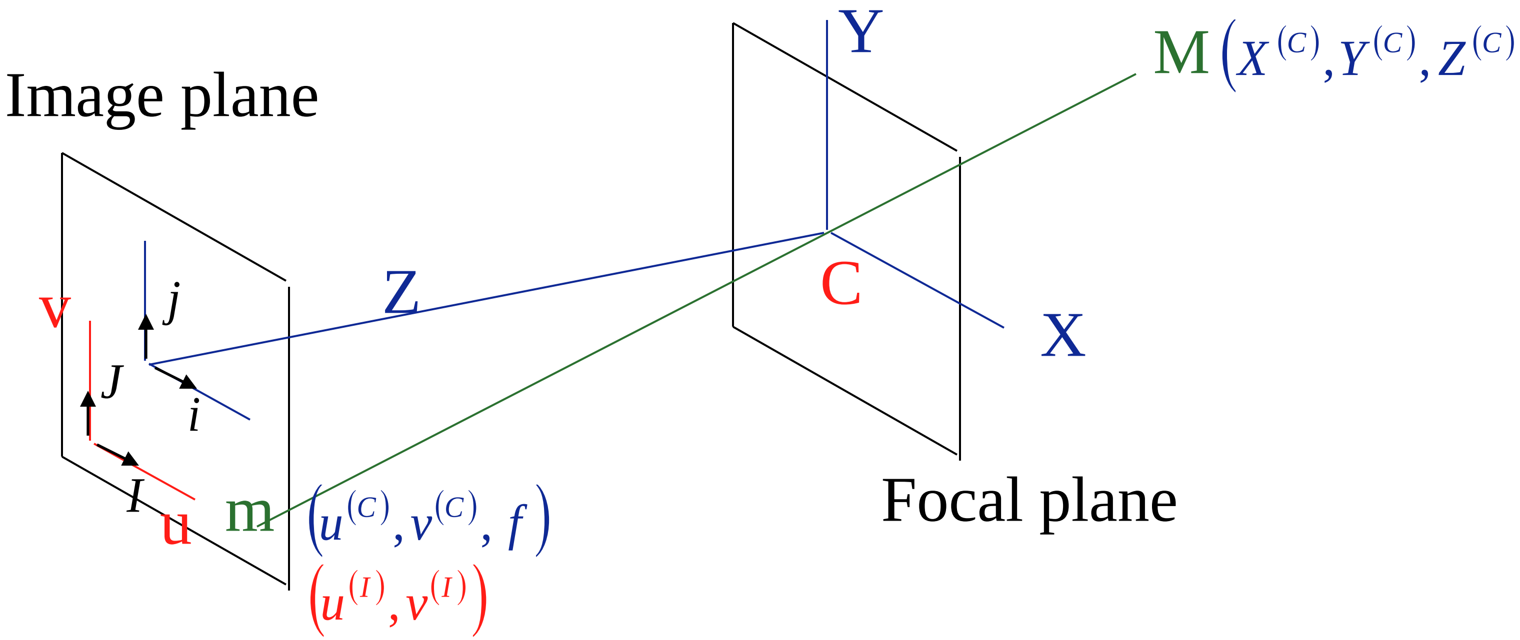


What if Camera Calibration is not known

Review: Intrinsic Camera Parameters

$$i = k_u I$$

$$j = k_v J$$



$$\begin{bmatrix} U^{(new)} \\ V^{(new)} \\ S \end{bmatrix} = \underbrace{\begin{bmatrix} -f_u & 0 & u_0 & 0 \\ 0 & -f_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{K} \begin{bmatrix} X^{(C)} \\ Y^{(C)} \\ Z^{(C)} \\ 1 \end{bmatrix}$$

$$f_u = f k_u$$

$$f_v = f k_v$$

Fundamental Matrix

$p^T \mathcal{E} p' = 0$ $\quad$ p and p' are in camera coordinate system

If u and u' are corresponding image coordinates then we have:

$$u = K_1 p \qquad p = K_1^{-1} u$$

$$u' = K_2 p' \qquad \Longrightarrow \qquad p' = K_2^{-1} u'$$

$$u^T K_1^{-T} \mathcal{E} K_2^{-1} u' = 0$$

$$\Rightarrow u^T F u' = 0$$

$$F = K_1^{-T} \mathcal{E} K_2^{-1}$$

Fundamental Matrix

$$u^T F u' = 0$$

$$F = K_1^{-T} \mathcal{E} K_2^{-1}$$

Fundamental Matrix is singular with rank 2.

In principal F has 7 parameters up to scale and can be estimated from 7 point correspondences.

Direct Simpler Method requires 8 correspondences (Olivier Faugeras,).

Estimating Fundamental Matrix

$$u^T F u' = 0$$

The 8-point algorithm (Faugeras)

Each point correspondence can be expressed as a linear equation:

$$\begin{bmatrix} u & v & 1 \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} uu' & uv' & u & u'v & vv' & v & u' & v' & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

The 8-point Algorithm

Scaling: Set F_{33} to 1 -> Solve for 8 parameters.

8 corresponding points, 8 equations.

$$\begin{pmatrix} u_1u_1' & u_1v_1' & u_1 & v_1u_1' & v_1v_1' & v_1 & u_1' & v_1' \\ u_2u_2' & u_2v_2' & u_2 & v_2u_2' & v_2v_2' & v_2 & u_2' & v_2' \\ u_3u_3' & u_3v_3' & u_3 & v_3u_3' & v_3v_3' & v_3 & u_3' & v_3' \\ u_4u_4' & u_4v_4' & u_4 & v_4u_4' & v_4v_4' & v_4 & u_4' & v_4' \\ u_5u_5' & u_5v_5' & u_5 & v_5u_5' & v_5v_5' & v_5 & u_5' & v_5' \\ u_6u_6' & u_6v_6' & u_6 & v_6u_6' & v_6v_6' & v_6 & u_6' & v_6' \\ u_7u_7' & u_7v_7' & u_7 & v_7u_7' & v_7v_7' & v_7 & u_7' & v_7' \\ u_8u_8' & u_8v_8' & u_8 & v_8u_8' & v_8v_8' & v_8 & u_8' & v_8' \end{pmatrix} \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \end{pmatrix} = - \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Invert and solve for $\mathcal{F}$.

(Use more points if available; find least-squares solution to minimize $\sum_{i=1}^{n} (\boldsymbol{p}_i^T \mathcal{F} \boldsymbol{p}_i')^2$)