Multi-View Geometry: Small Motion (Chapter 7 and 11 Szelisky)

Guido Gerig

CS 6320 Spring 2012

Small Motions and Epipolar Constraint

Motion Models (Review)

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} \approx \begin{bmatrix} 1 & -\gamma & \beta \\ \gamma & 1 & -\alpha \\ -\beta & \alpha & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}$$

3D Rigid Motion

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} \approx \left(\begin{bmatrix} 0 & -\gamma & \beta \\ \gamma & 0 & -\alpha \\ -\beta & \alpha & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}$$

$$\begin{bmatrix} X' - X \\ Y' - Y \\ Z' - Z \end{bmatrix} \approx \begin{bmatrix} 0 & -\gamma & \beta \\ \gamma & 0 & -\alpha \\ -\beta & \alpha & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}$$

$$\begin{bmatrix} V_X \\ V_Y \\ V_Z \end{bmatrix} \approx \begin{bmatrix} 0 & -\omega_Z & \omega_Y \\ \omega_Z & 0 & -\omega_X \\ -\omega_Y & \omega_X & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} V_{T_x} \\ V_{T_Y} \\ V_{T_Z} \end{bmatrix}$$

$\begin{bmatrix} V_X \\ V_Y \\ V_Z \end{bmatrix} =$ Velocity Vector

$\begin{bmatrix} V_{T_X} \\ V_{T_Y} \\ V_{T_Z} \end{bmatrix} =$ Translational Component of Velocity

$\begin{bmatrix} \omega_X \\ \omega_Y \\ \omega_Z \end{bmatrix} =$ Angular Velocity

Small Motions

$$\dot{p} = \begin{bmatrix} V_X \\ V_Y \\ V_Z \end{bmatrix} = \text{ Velocity Vector}$$

$$v = \begin{bmatrix} V_{T_X} \\ V_{T_Y} \\ V_{T_Z} \end{bmatrix} = \text{ Translational Component of Velocity}$$

$$\omega = \begin{bmatrix} \omega_X \\ \omega_Y \\ \omega_Z \end{bmatrix} = \text{ Angular Velocity}$$

$$t = \delta t v$$

$$R = I + \delta t[\omega_\times]$$

$$p' = p + \delta t \dot{p}$$

$$p^T \mathcal{E} p' = 0$$

$$p^T [v_\times](I + \delta t[\omega_\times])(p + \delta t \dot{p}) = 0$$

$$p^T ([v_\times][\omega_\times])p - (p \times \dot{p}).v = 0$$

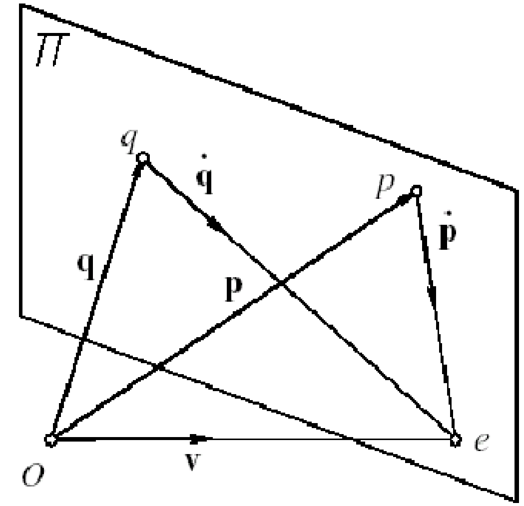
Translating Camera

$$p^T([v_\times][\omega_\times])p - (p \times \dot{p}).v =$$

$$\omega = 0$$

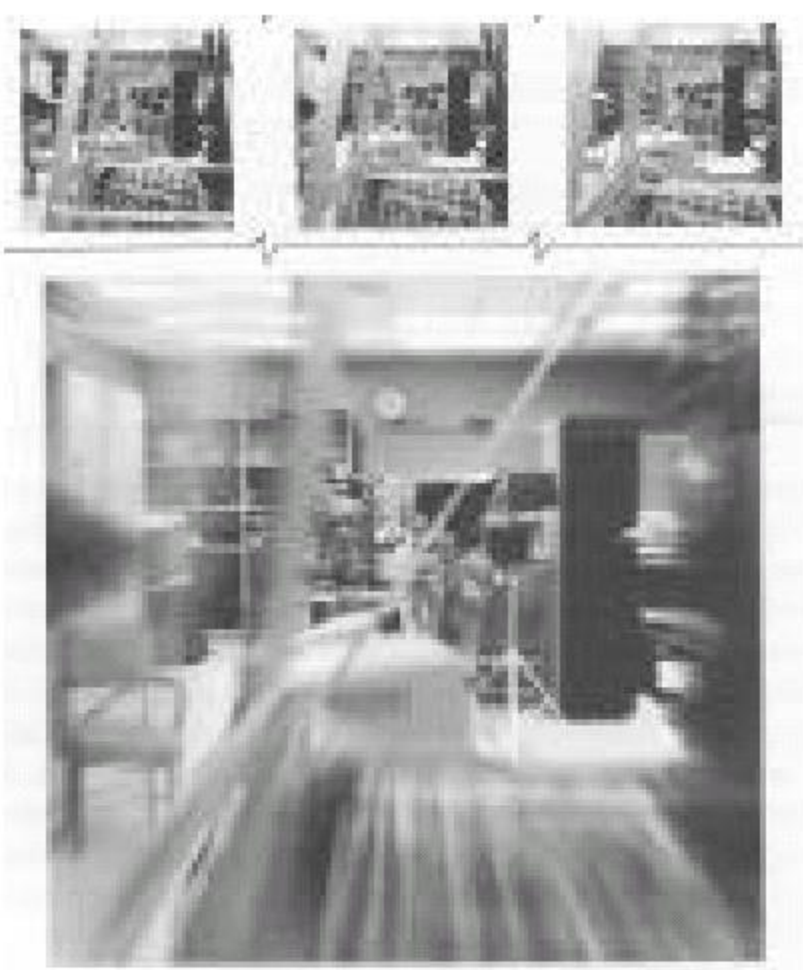
$$(p \times \dot{p}).v = 0$$

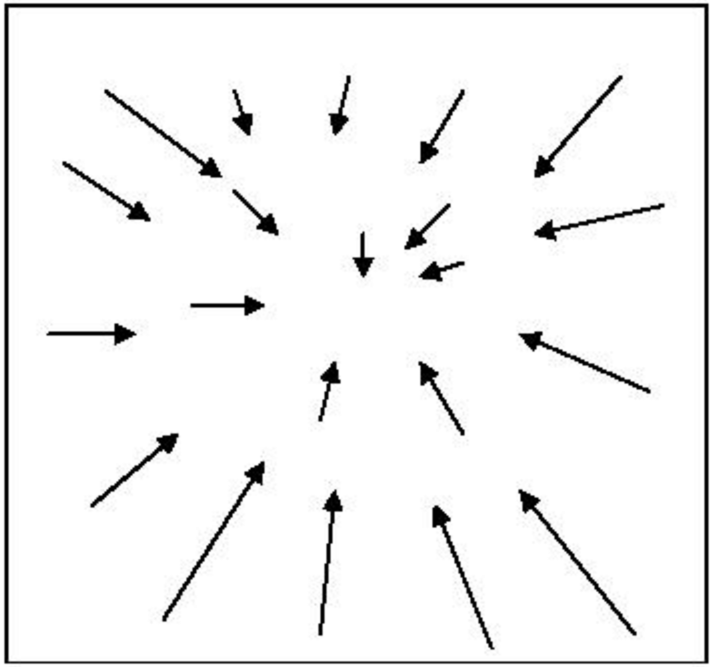
p, $\dot{p}$, and v are coplanar



Focus of expansion (FOE): Under pure translation, the motion field at every point in the image points toward the focus of expansion

FOE for Translating Camera





FOE from Basic Equations of Motion

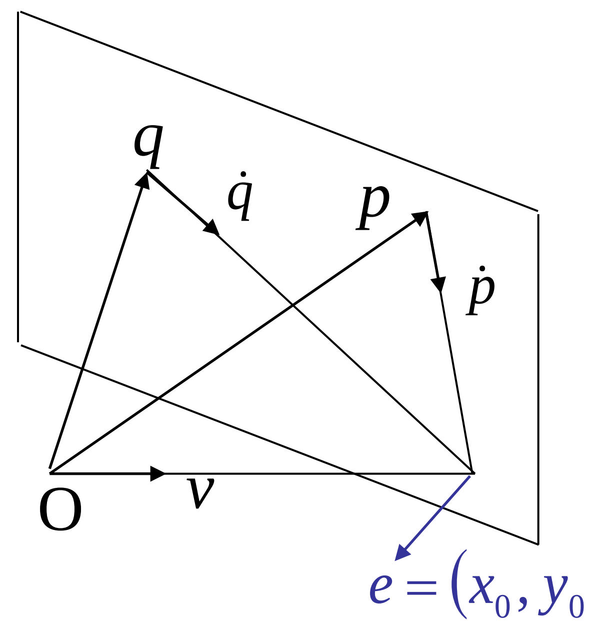
$$\dot{p}_x = \frac{V_{T_Z} x - V_{T_X} f}{Z} - \omega_Y f + \omega_Z y + \frac{\omega_X xy}{f} - \frac{\omega_Y x^2}{f}$$

$$\dot{p}_y = \frac{V_{T_Z} y - V_{T_Y} f}{Z} + \omega_X f - \omega_Z x - \frac{\omega_Y xy}{f} + \frac{\omega_X y^2}{f}$$

$$\omega = 0$$

$$\dot{p}_x = \frac{V_{T_Z} x - V_{T_X} f}{Z}$$

$$\dot{p}_y = \frac{V_{T_Z} y - V_{T_Y} f}{Z}$$



$$x_0 = f \frac{V_{T_X}}{V_{T_Z}}$$

$$y_0 = f \frac{V_{T_Y}}{V_{T_Z}}$$

$$\Longrightarrow$$

$$\dot{p}_x = (x - x_0)\frac{V_{T_Z}}{Z}$$

$$\dot{p}_y = (y - y_0)\frac{V_{T_Z}}{Z}$$