Multi-View Geometry Part II (Ch7 New book. Ch 10/11 old book)

Guido Gerig

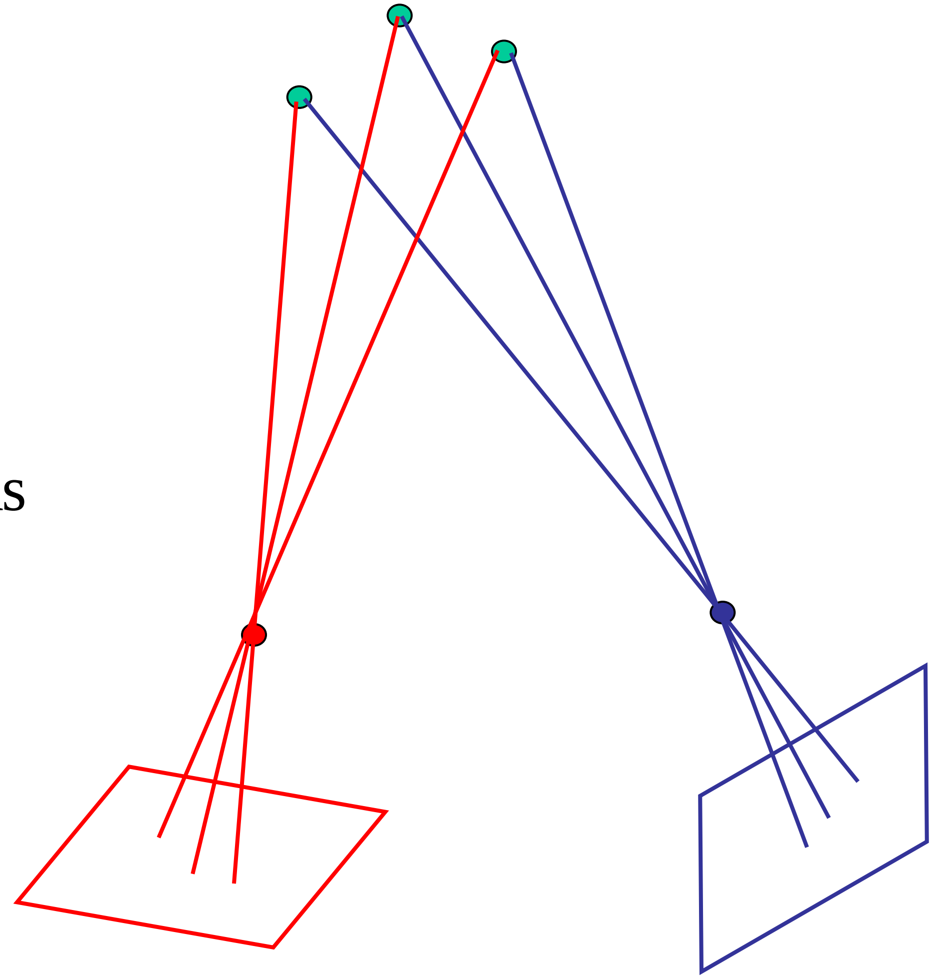
CS 6320 Spring 2012

Credits: M. Shah, UCF CAP5415, lecture 23
http://www.cs.ucf.edu/courses/cap6411/cap5415/, Trevor Darrell, Berkeley, C280, Marc Pollefeys

Multi-View Geometry

Relates

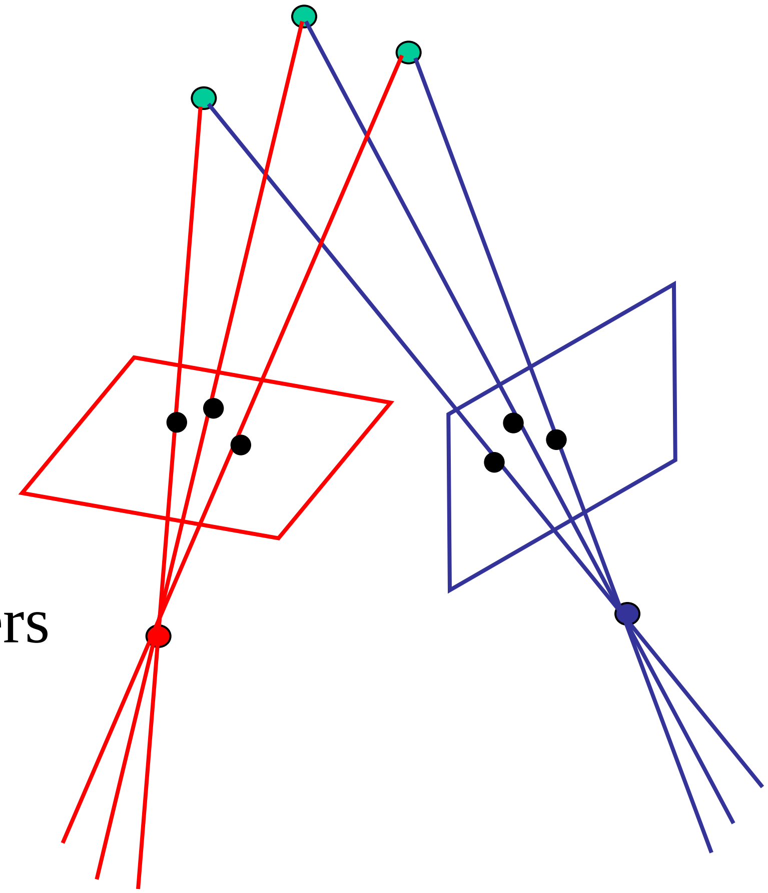
- 3D World Points
- Camera Centers
- Camera Orientations



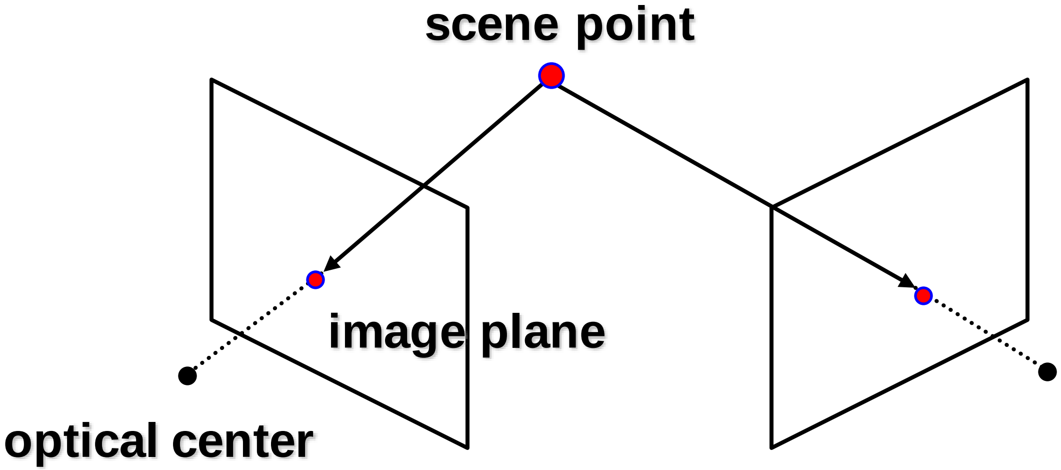
Multi-View Geometry

Relates

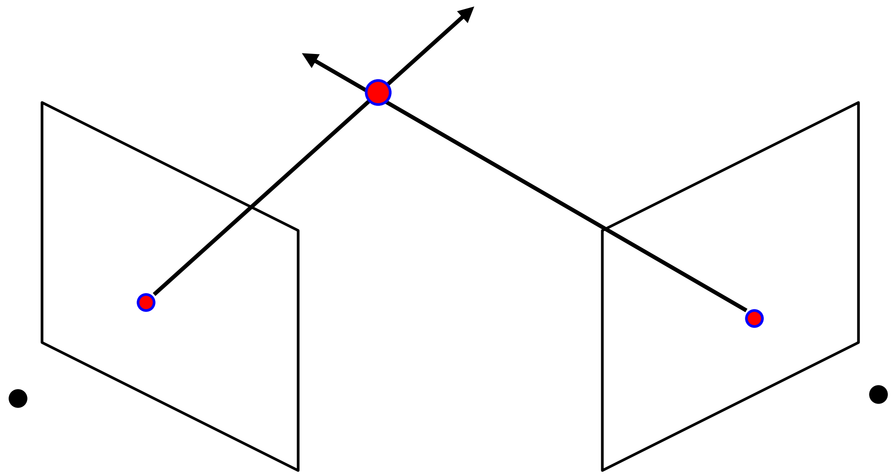
- 3D World Points
- Camera Centers
- Camera Orientations
- Camera Intrinsic Parameters
- Image Points



Stereo



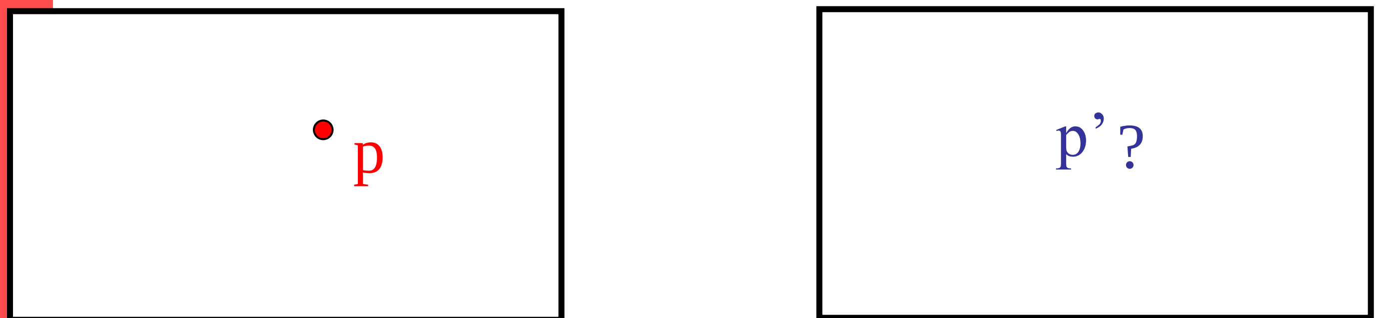
Stereo



Basic Principle: Triangulation

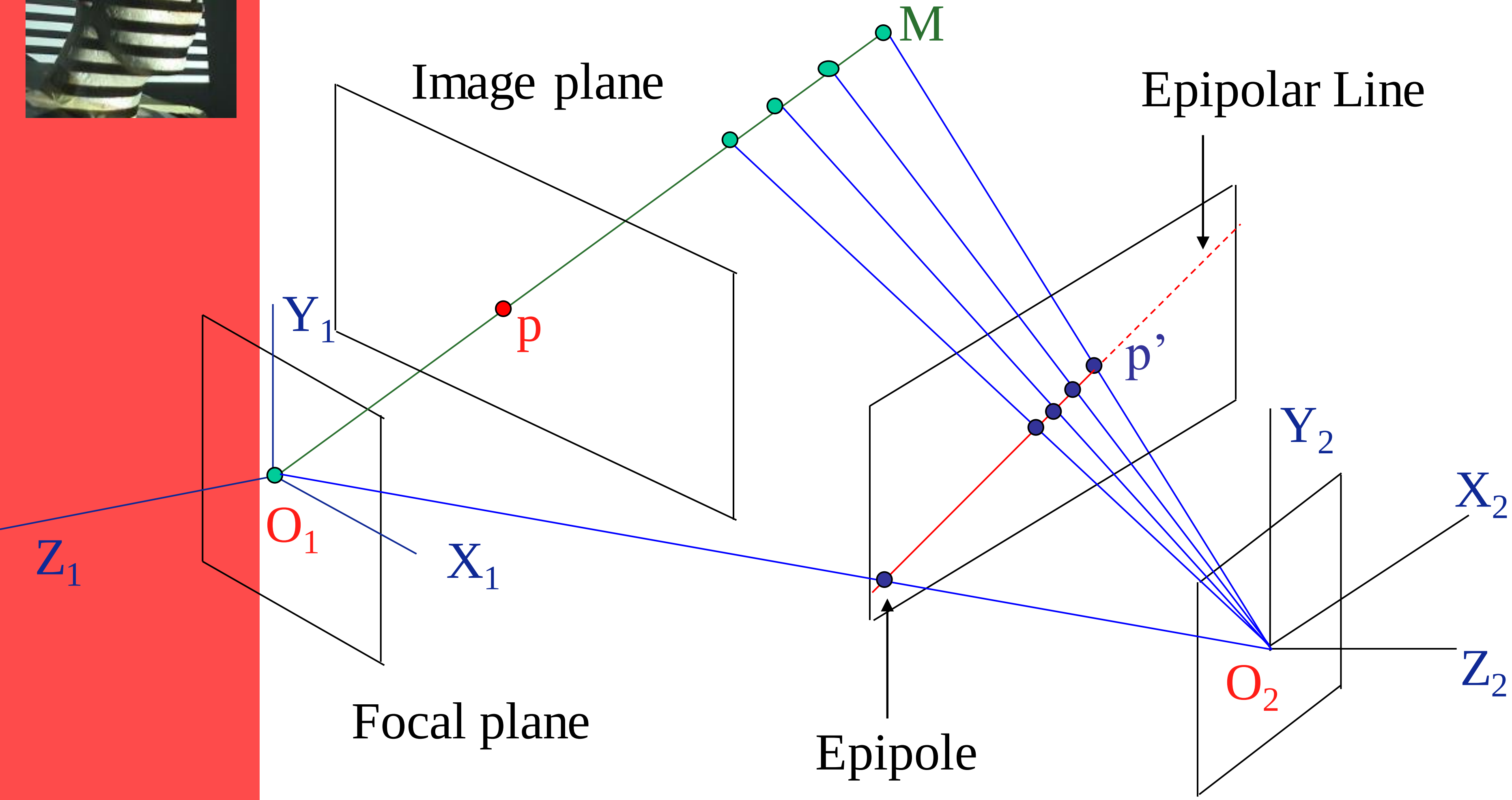
- Gives reconstruction as intersection of two rays
- Requires
  - calibration
  - ***point correspondence***

Stereo Constraints



Given p in left image, where can the corresponding point p' in right image be?

Stereo Constraints



Demo Epipolar Geometry

Java Applet

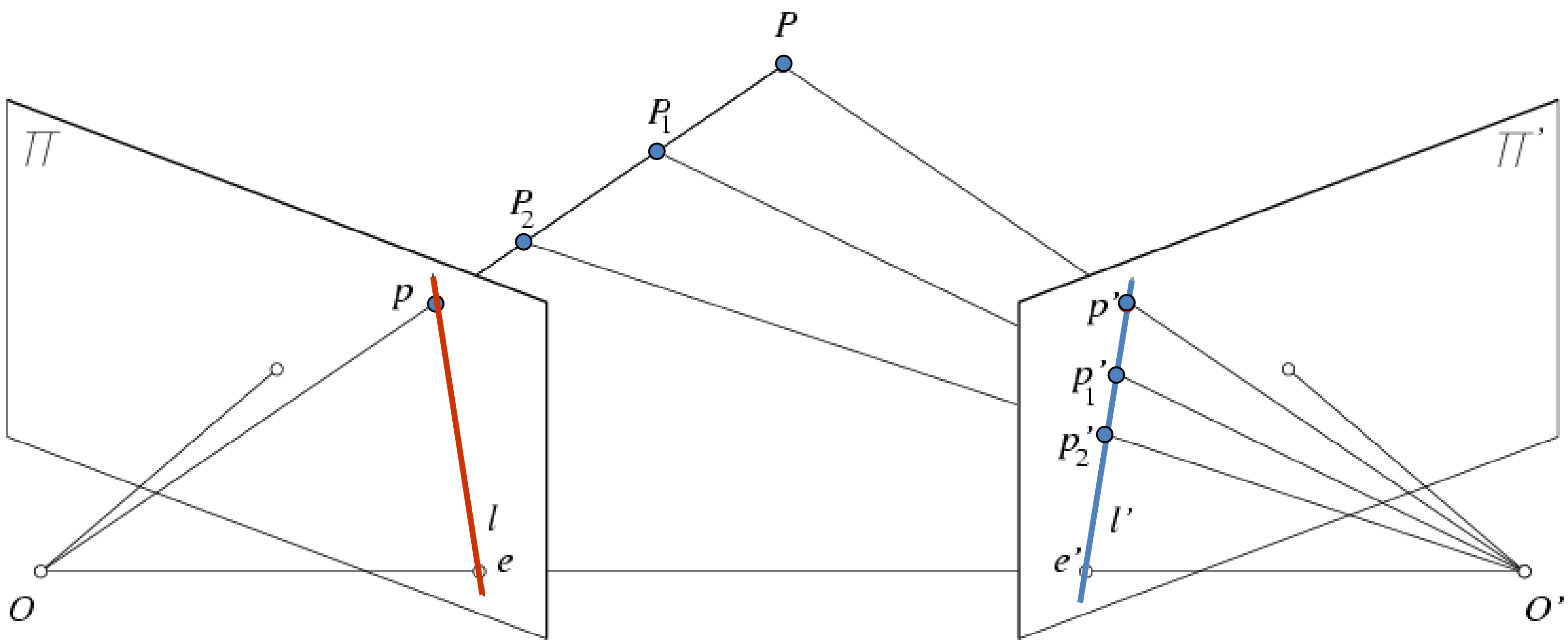
credit to:

Quang-Tuan Luong

SRI Int.

Sylvain Bougnoux

Epipolar constraint



- Potential matches for *p* have to lie on the corresponding epipolar line *l'*.
- Potential matches for *p'* have to lie on the corresponding epipolar line *l*.

http://www.ai.sri.com/~luong/research/Meta3DViewer/EpipolarGeo.html

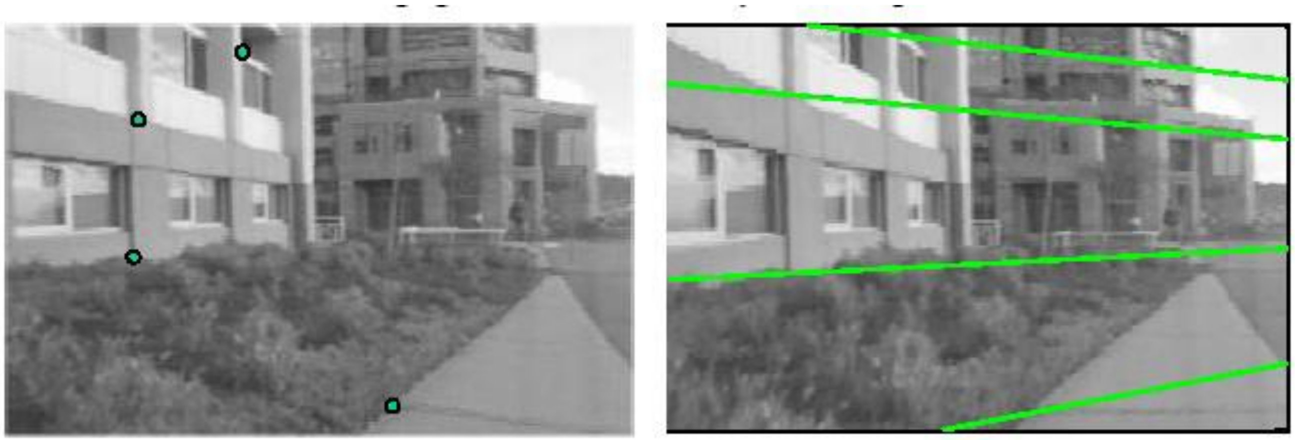
Source: M. Pollefeys

Finding Correspondences

Andrea Fusiello, CVonline

Strong constraints for searching for corresponding points!

Example



Parallel Cameras: Corresponding points on horizontal lines.

Epipolar Constraint

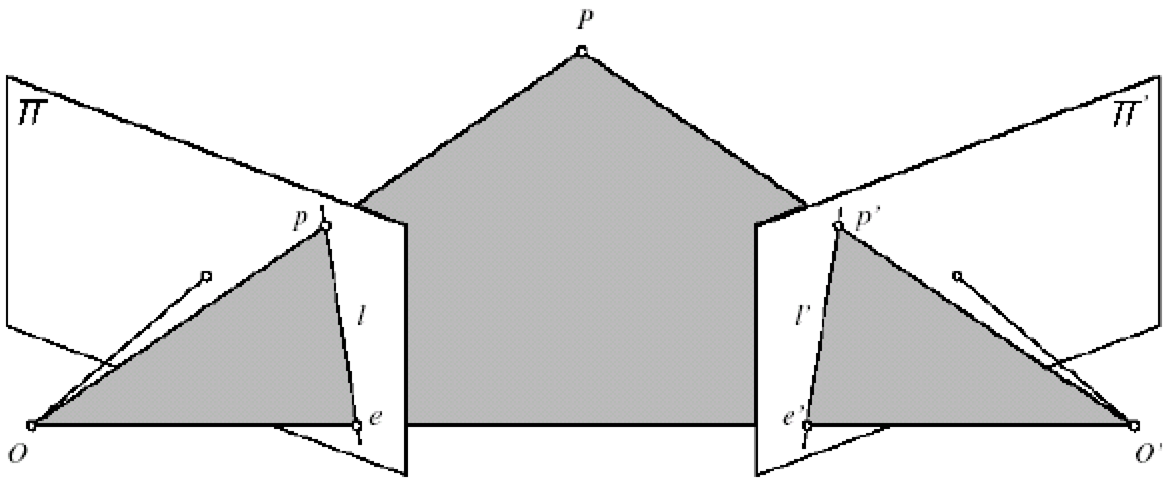


FIGURE 11.1: Epipolar geometry: the point P, the optical centers O and O' of the two cameras, and the two images p and p' of P all lie in the same plane.

All epipolar lines contain epipole, the image of other camera center.

From Geometry to Algebra

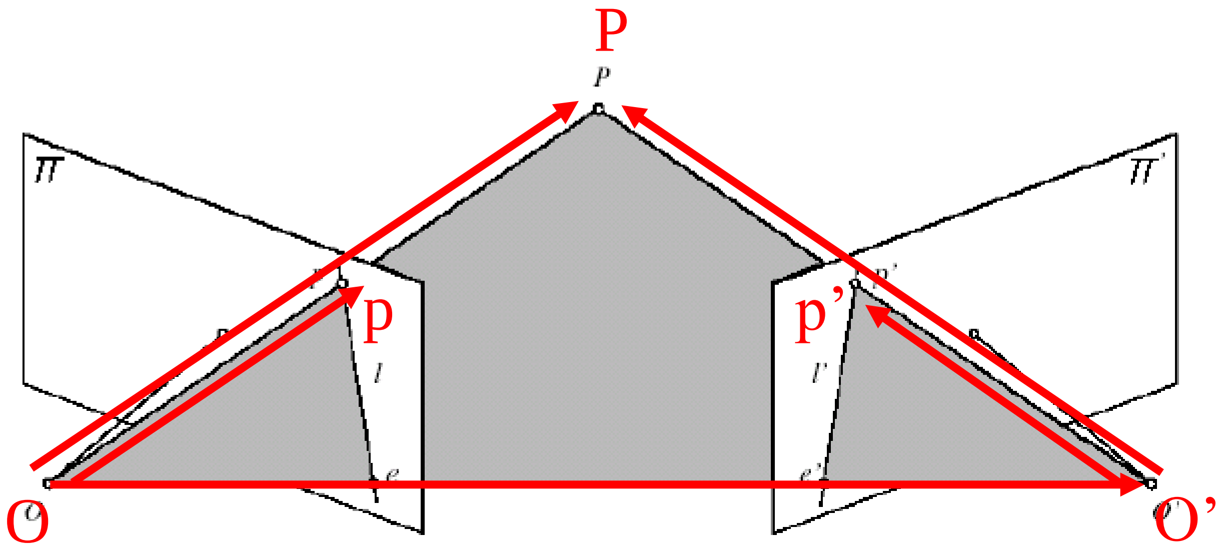
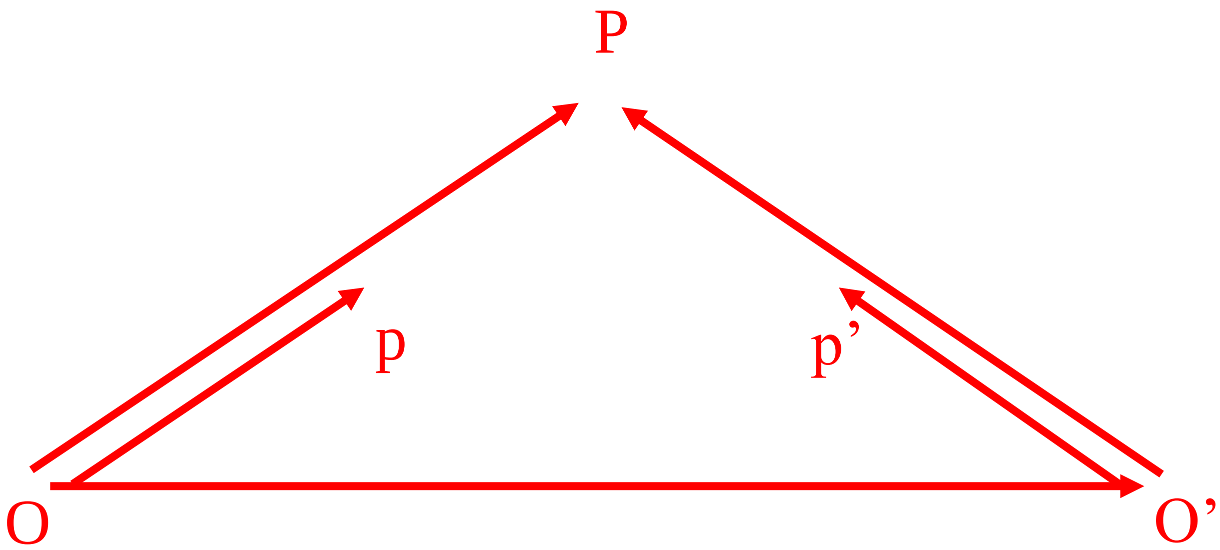


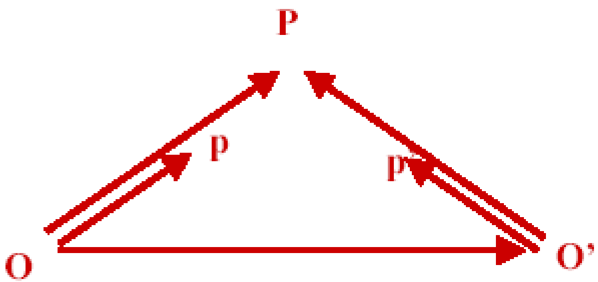
FIGURE 11.1: Epipolar geometry: the point P, the optical centers O and O' of the two cameras, and the two images p and p' of P all lie in the same plane.

From Geometry to Algebra

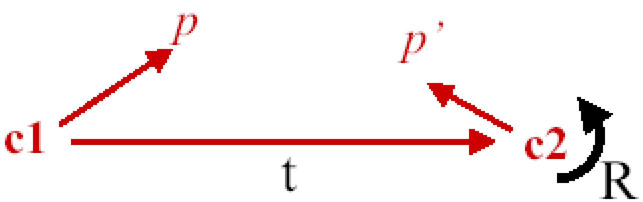


The epipolar constraint: these vectors are coplanar:

$$\overrightarrow{Op} \cdot [\overrightarrow{OO'} \times \overrightarrow{O'p'}] = 0$$



$$\overrightarrow{Op}\cdot[\overrightarrow{OO'}\times\overrightarrow{O'p'}]=0$$



p,p' are image coordinates of P in c1 and c2…

c2 is related to c1 by rotation R and translation t

$$\boldsymbol{p}\cdot[\boldsymbol{t}\times(\mathcal{R}\boldsymbol{p}')]=0$$

Linear Constraint:
Should be able to express as matrix multiplication.

Review: Matrix Form of Cross Product

The vector cross product also acts on two vectors and returns a third vector. Geometrically, this new vector is constructed such that its projection onto either of the two input vectors is zero.

$$\vec{a} \times \vec{b} = \begin{bmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{bmatrix}$$

$$\vec{a} \times \vec{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = \vec{c}$$

$$\vec{a} \cdot \vec{c} = 0$$

$$\vec{b} \cdot \vec{c} = 0$$

Review: Matrix Form of Cross Product

$$\vec{a} \times \vec{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = \vec{c}$$

$$\vec{a} \cdot \vec{c} = 0$$

$$\vec{b} \cdot \vec{c} = 0$$

$$[a_x] = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

$$\vec{a} \times \vec{b} = [a_x]\vec{b}$$

Matrix Form

$$\boldsymbol{p} \cdot [\boldsymbol{t} \times (\mathcal{R}\boldsymbol{p}')] = 0$$

$$\vec{a} \times \vec{b} = [a_x]\vec{b}$$

$$p^T [t_x] \Re p' = 0$$

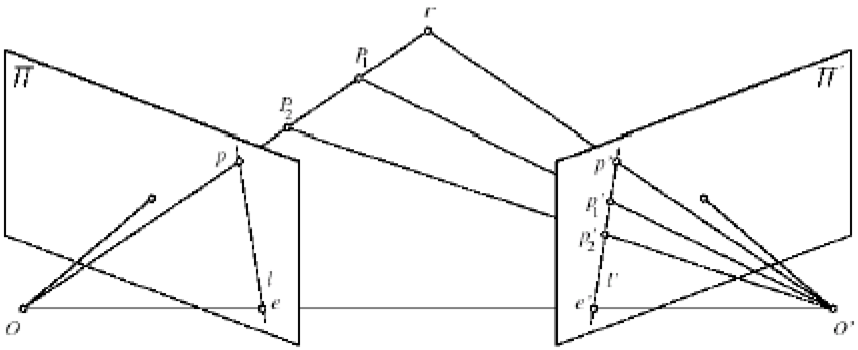
$$\varepsilon = [t_x] \Re$$

$$\boldsymbol{p}^T \mathcal{E} \boldsymbol{p}' = 0$$

The Essential Matrix

Matrix that relates image of point in one camera to a second camera, given translation and rotation.

$$\mathcal{E} = [t_x]\mathfrak{R}$$



$$\mathbf{p}^T \mathcal{E} \mathbf{p}' = 0$$

$$\vec{a} \times \vec{b} = [a_x]\vec{b}$$

The Essential Matrix

- Based on the Relative Geometry of the Cameras
- Assumes Cameras are calibrated (i.e., intrinsic parameters are known)
- Relates image of point in one camera to a second camera (points in camera coordinate system).
- Is defined up to scale
- 5 independent parameters

The Essential Matrix

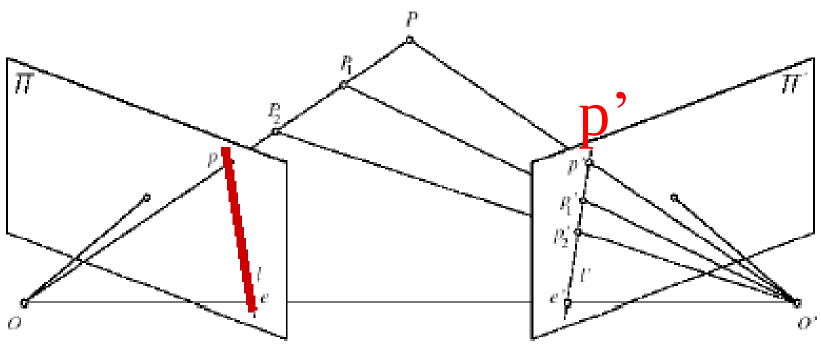
$$p^T \mathcal{E} p' = 0$$

What is $\mathcal{E}p'$?

The Essential Matrix

$\mathcal{E}p'$ is the epipolar line corresponding to p’ in the left camera.

$$au + bv + c = 0$$



$\mathcal{E}p'$

$$p = (u, v, 1)^T$$

$$l = (a, b, c)^T$$

$$l \cdot p = 0$$

$$\mathcal{E}p' \cdot p = 0$$

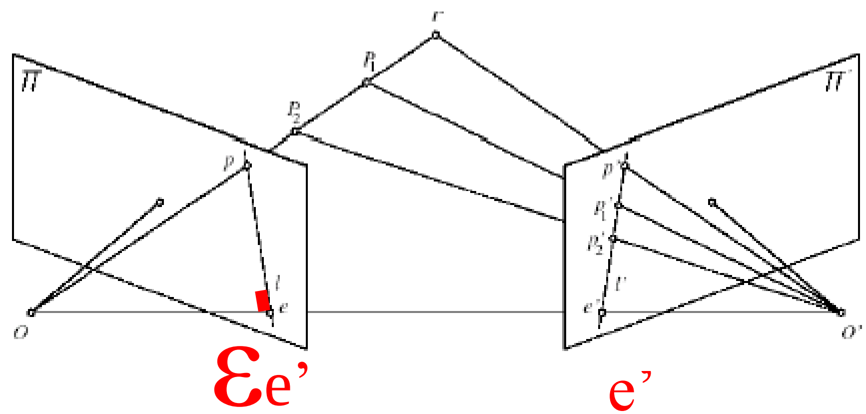
$$\mathbf{p}^T \mathcal{E} \mathbf{p}' = 0$$

Similarly $\mathcal{E}^T p$ is the epipolar line corresponding to **p** in the right camera.

The Essential Matrix

$e^T \mathcal{E} e'$?

What is $\mathcal{E} e'$?



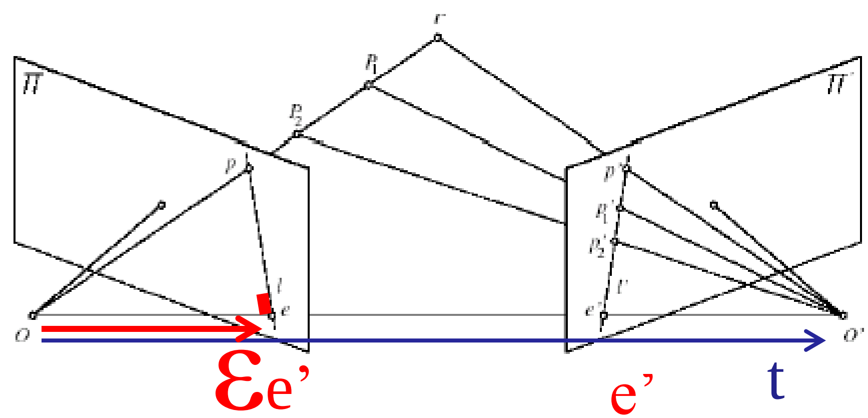
- line $\mathcal{E} p'$ converges to epipole e
- e' expressed in frame C_1

The Essential Matrix

$\mathcal{E}e' = [t_\times]\mathrm{R}e' = 0$ (e' in frame C_1 parallel to t)

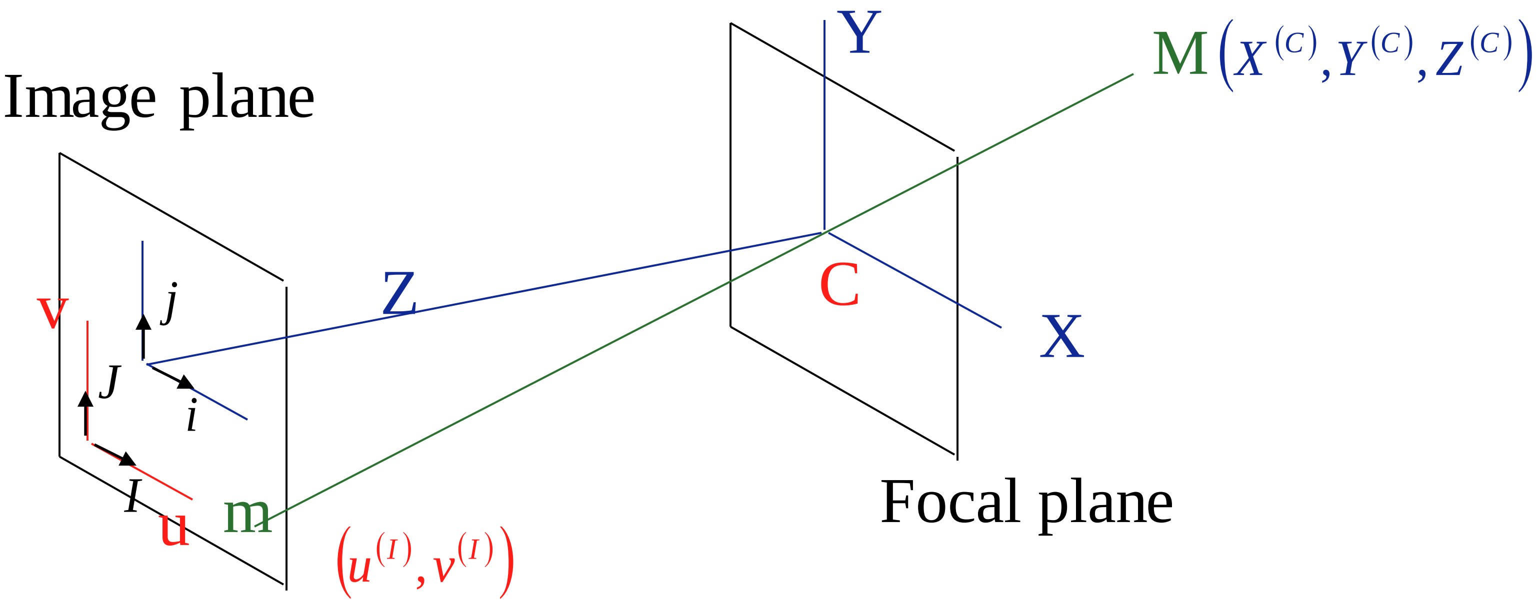
Similarly, $\mathcal{E}^T e = R^T [t_\times]^T e = -R^T [t_\times] e = 0$

Essential Matrix is singular with rank 2



What if Camera Calibration is not known

Review: Intrinsic Camera Parameters



$$\begin{bmatrix} u^{(I)} \\ v^{(I)} \\ S \end{bmatrix} = \underbrace{\begin{bmatrix} -f_u & 0 & u_0 & 0 \\ 0 & -f_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{K} \begin{bmatrix} X^{(C)} \\ Y^{(C)} \\ Z^{(C)} \\ 1 \end{bmatrix}$$

$$f_u = fk_u = \alpha$$
$$f_v = fk_v = \beta$$
$$\Theta = 90^\circ$$

Fundamental Matrix

$p^T \mathcal{E} p' = 0$ p and p' are in camera coordinate system

If u and u' are corresponding image coordinates then we have:

$$u = K_1 p \qquad p = K_1^{-1} u \rightarrow p^T = \left(K_1^{-1} u\right)^T = u^T K_1^{-T}$$

$$u' = K_2 p' \Longrightarrow p' = K_2^{-1} u'$$

$$u^T K_1^{-T} \mathcal{E} K_2^{-1} u' = 0$$

$$\Rightarrow u^T F u' = 0$$

$$F = K_1^{-T} \mathcal{E} K_2^{-1}$$

Fundamental Matrix

$$u^T F u' = 0$$

$$F = K_1^{-T} \mathcal{E} K_2^{-1}$$

Fundamental Matrix is singular with rank 2.

In principal F has 7 parameters up to scale and can be estimated from 7 point correspondences.

Direct Simpler Method requires 8 correspondences (Olivier Faugeras,, Computer Vision textbook).

Estimating Fundamental Matrix

$$u^T F u' = 0$$

The 8-point algorithm (Faugeras)

Each point correspondence can be expressed as a linear equation:

$$\begin{bmatrix} u & v & 1 \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} uu' & uv' & u & u'v & vv' & v & u' & v' & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

The 8-point Algorithm

Scaling: Set F_{33} to 1 -> Solve for 8 parameters.

8 corresponding points, 8 equations.

$$\begin{pmatrix} u_1u'_1 & u_1v'_1 & u_1 & v_1u'_1 & v_1v'_1 & v_1 & u'_1 & v'_1 \\ u_2u'_2 & u_2v'_2 & u_2 & v_2u'_2 & v_2v'_2 & v_2 & u'_2 & v'_2 \\ u_3u'_3 & u_3v'_3 & u_3 & v_3u'_3 & v_3v'_3 & v_3 & u'_3 & v'_3 \\ u_4u'_4 & u_4v'_4 & u_4 & v_4u'_4 & v_4v'_4 & v_4 & u'_4 & v'_4 \\ u_5u'_5 & u_5v'_5 & u_5 & v_5u'_5 & v_5v'_5 & v_5 & u'_5 & v'_5 \\ u_6u'_6 & u_6v'_6 & u_6 & v_6u'_6 & v_6v'_6 & v_6 & u'_6 & v'_6 \\ u_7u'_7 & u_7v'_7 & u_7 & v_7u'_7 & v_7v'_7 & v_7 & u'_7 & v'_7 \\ u_8u'_8 & u_8v'_8 & u_8 & v_8u'_8 & v_8v'_8 & v_8 & u'_8 & v'_8 \end{pmatrix} \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \end{pmatrix} = - \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Invert and solve for $\mathcal{F}$.

(Use more points if available; find least-squares solution to minimize $\sum_{i=1}^{n} (\boldsymbol{p}_i^T \mathcal{F} \boldsymbol{p}'_i)^2$)