



Keypoint Detection: Harris Operator

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Keypoint Detection

- Where will the interest points come from?
 - What are salient features that we'll *detect* in multiple views?
- How to *describe* a local region?
- How to establish *correspondences*, i.e., compute matches?

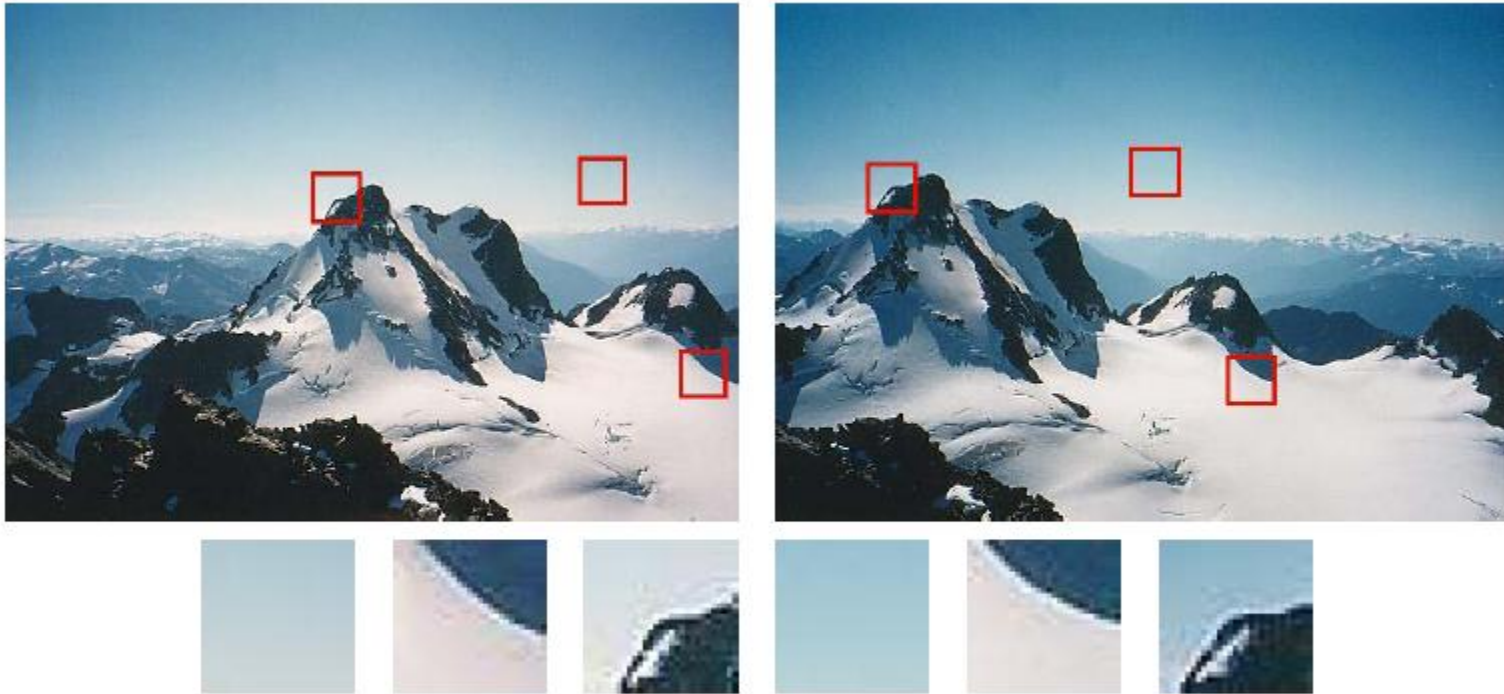
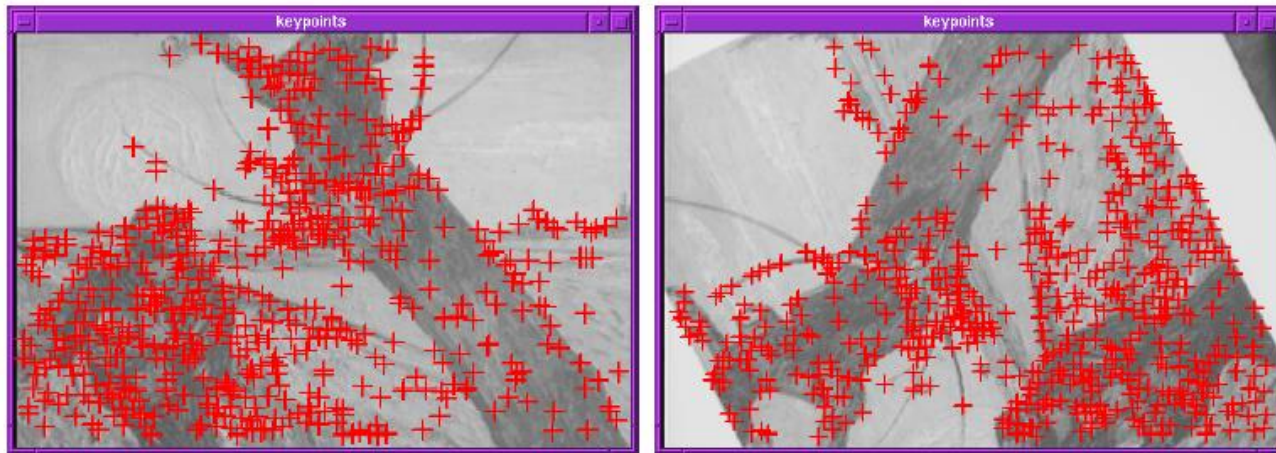


Figure 4.3: *Image pairs with extracted patches below. Notice how some patches can be localized or matched with higher accuracy than others.*

Finding Corners



Key property: in the region around a corner, image gradient has two or more dominant directions

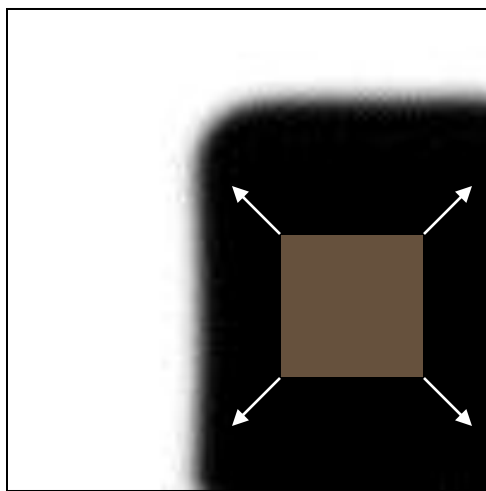
Corners are repeatable and **distinctive**

C.Harris and M.Stephens. ["A Combined Corner and Edge Detector."](#)
Proceedings of the 4th Alvey Vision Conference: pages 147--151.

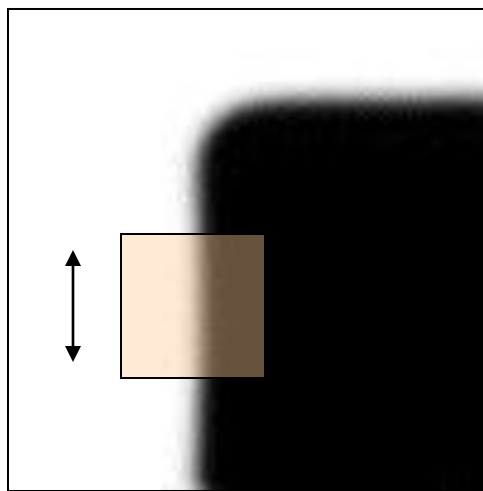
Corners as distinctive interest points

We should easily recognize the point by looking through a small window

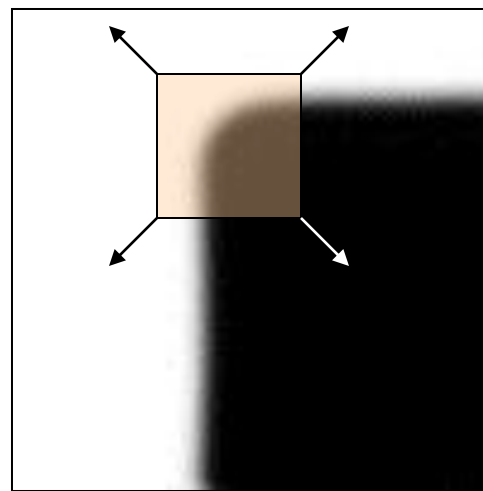
Shifting a window in *any direction* should give *a large change* in intensity



“flat” region:
no change in
all directions

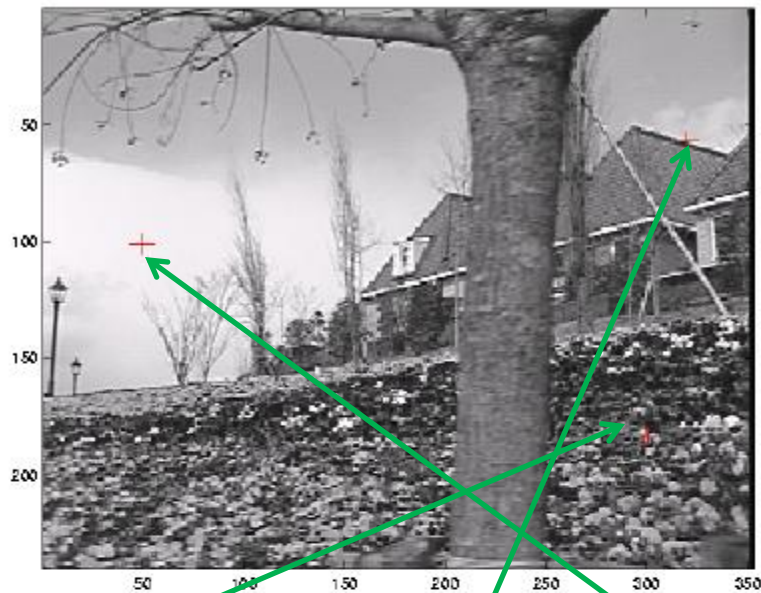


“edge”:
no change
along the edge
direction

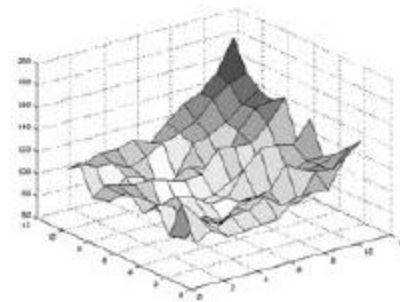
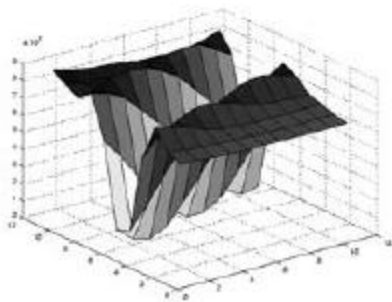
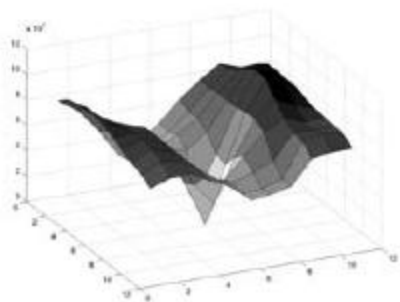
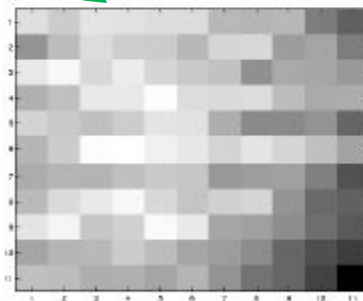
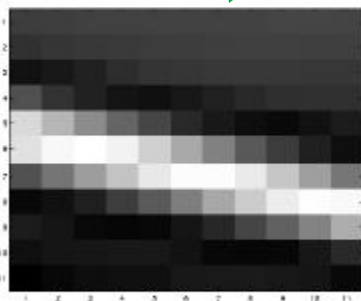
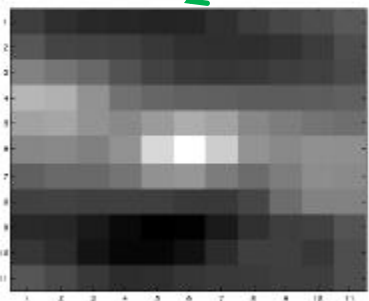


“corner”:
significant
change in all
directions

Local neighborhoods of 3 distinctive local patterns: Strong point features (points, corners) represent good landmarks.



(a)



Harris Detector formulation

Change of intensity for the shift $[u, v]$:

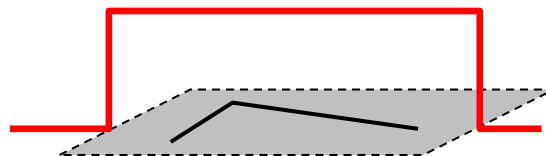
$$E(u, v) = \sum_{x, y} w(x, y) [I(x+u, y+v) - I(x, y)]^2$$

Window function

Shifted intensity

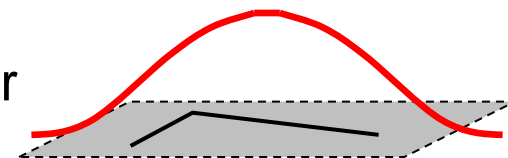
Intensity

Window function $w(x, y) =$



1 in window, 0 outside

or



Gaussian

Harris Detector formulation

This measure of change can be approximated by:

$$E(u, v) \approx [u \ v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a 2×2 matrix computed from image derivatives:

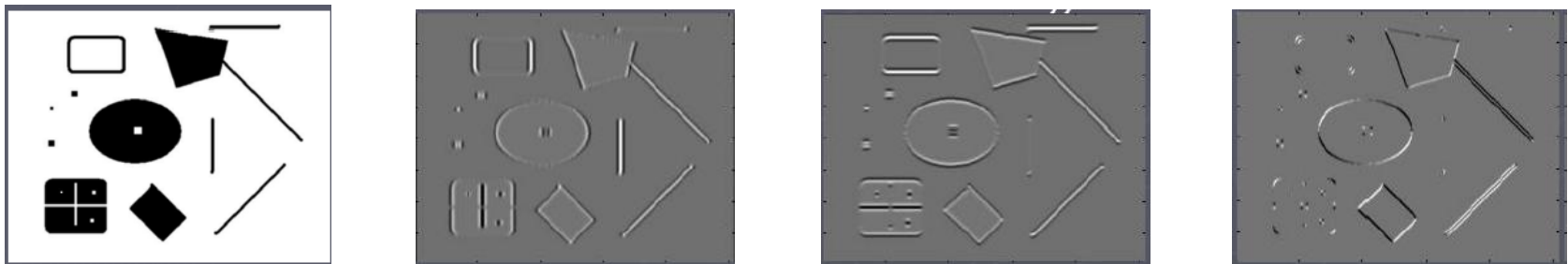
$$M = \sum_{x,y} w(x, y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Gradient with respect to x , times gradient with respect to y

Sum over image region – area we are checking for corner

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \ I_y]$$

Harris Detector formulation



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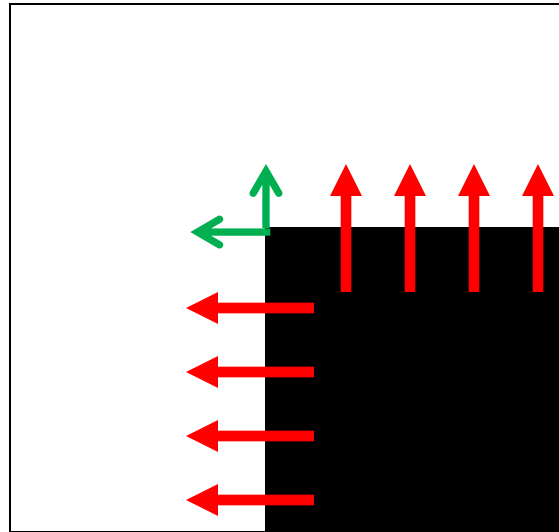
Gradient with respect to x , times gradient with respect to y

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What does this matrix reveal?

First, consider an axis-aligned corner:



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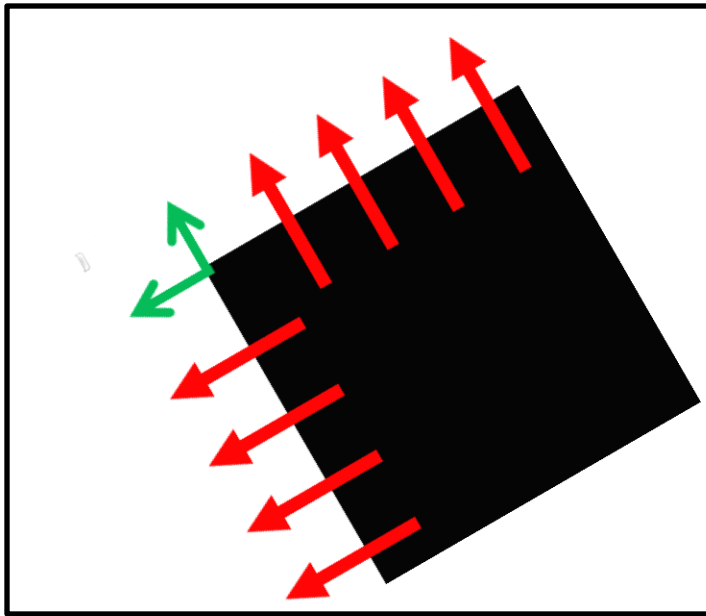
$$M = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

This means dominant gradient directions align with x or y axis

If either λ is close to 0, then this is **not** a corner, so look for locations where both are large.

What if we have a corner that is not aligned with the image axes?

Arbitrary rotation



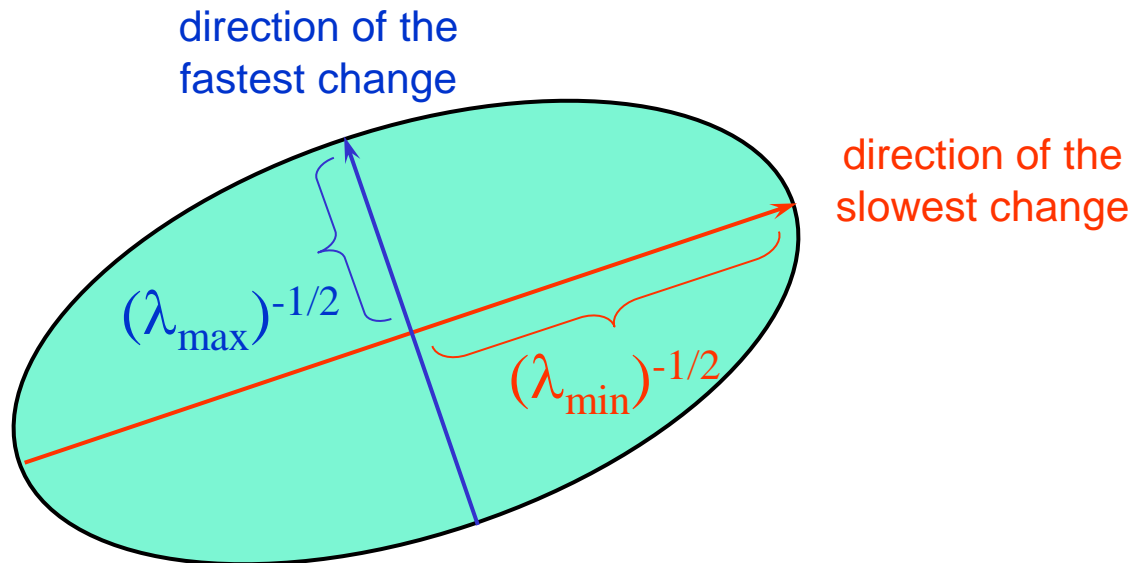
$$M = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$$

- M arbitrary, positive semidefinite
- Edges and corner no more aligned with image axes

General Case

Since M is symmetric, we have $M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$
(eigenvalue-eigenvector transformation)

We can visualize M as an ellipse with axis lengths determined by the eigenvalues and orientation determined by R



Interpreting the eigenvalues

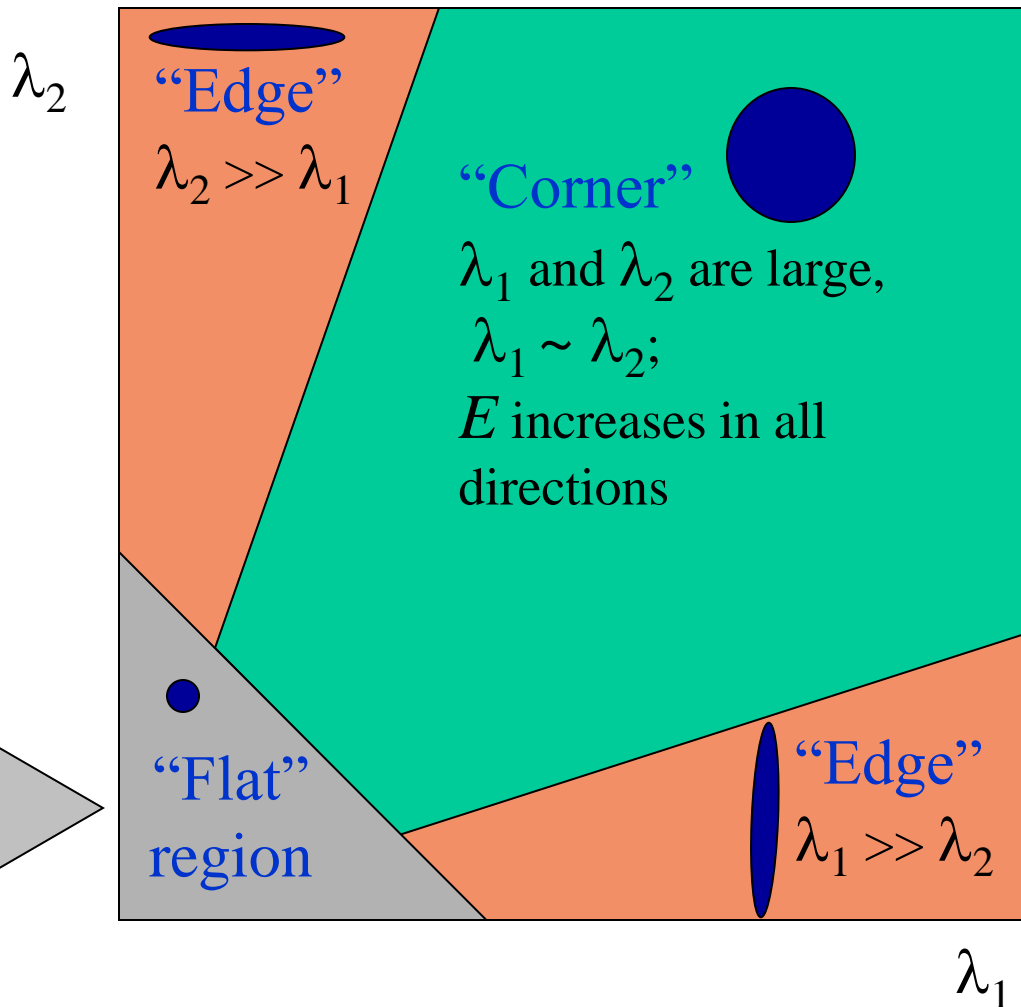
Classification of image points using eigenvalues of M :

Edge (strong response across edge, small response along edge):
 $\lambda_1 \gg \lambda_2$ or $\lambda_2 \gg \lambda_1$

Corner (strong responses in both directions):
 λ_1 and $\lambda_2 \gg 0$

Flat (small or no response in both directions):
 λ_1 and λ_2 small

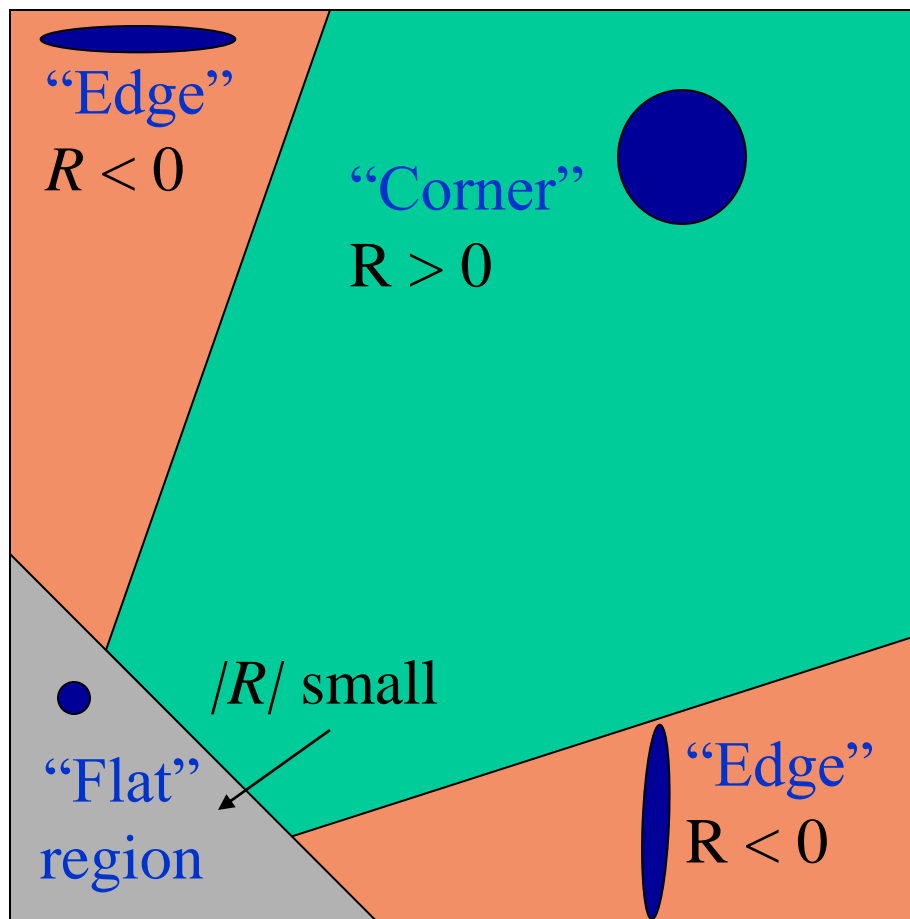
λ_1 and λ_2 are small;
 E is almost constant in all directions



Corner response function: Harris

$$\text{Response} = \det(M) - \alpha \text{trace}(M)^2 = \lambda_1 \lambda_2 - \alpha (\lambda_1 + \lambda_2)^2$$

α : constant (0.04 to 0.06)



Harris Corner Detector

- Algorithm steps:
 - Compute M matrix within all image windows to get their Response scores
 - Find points with large corner response
(*Response* > threshold)
 - Take the points of local maxima of *Response*
(*search local neighborhoods, e.g. 3x3 or 5x5 for location of maximum response*).

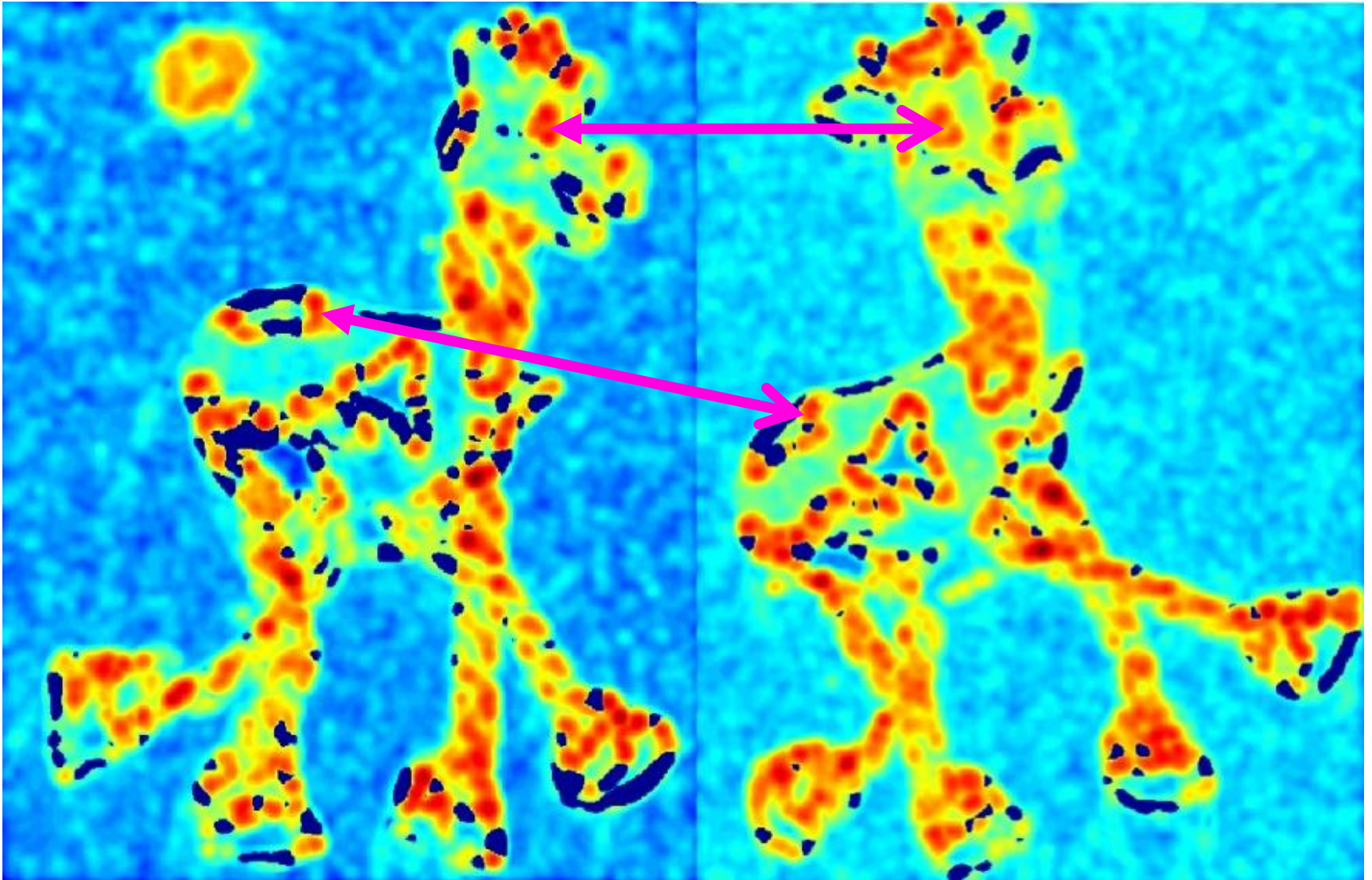
Harris Detector: Workflow



Slide adapted form Darya Frolova, Denis Simakov, Weizmann Institute.

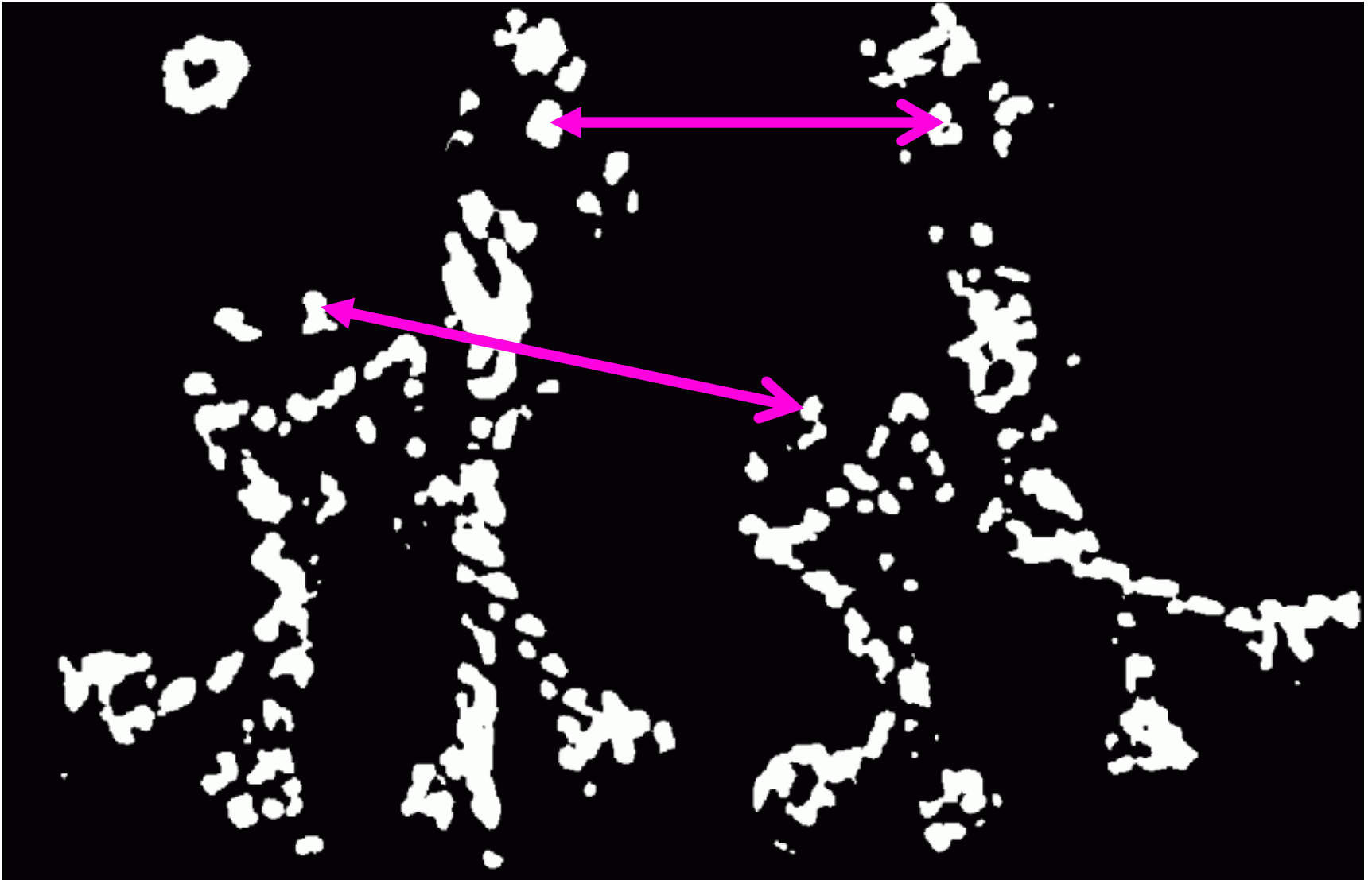
Harris Detector: Workflow

Compute corner response R



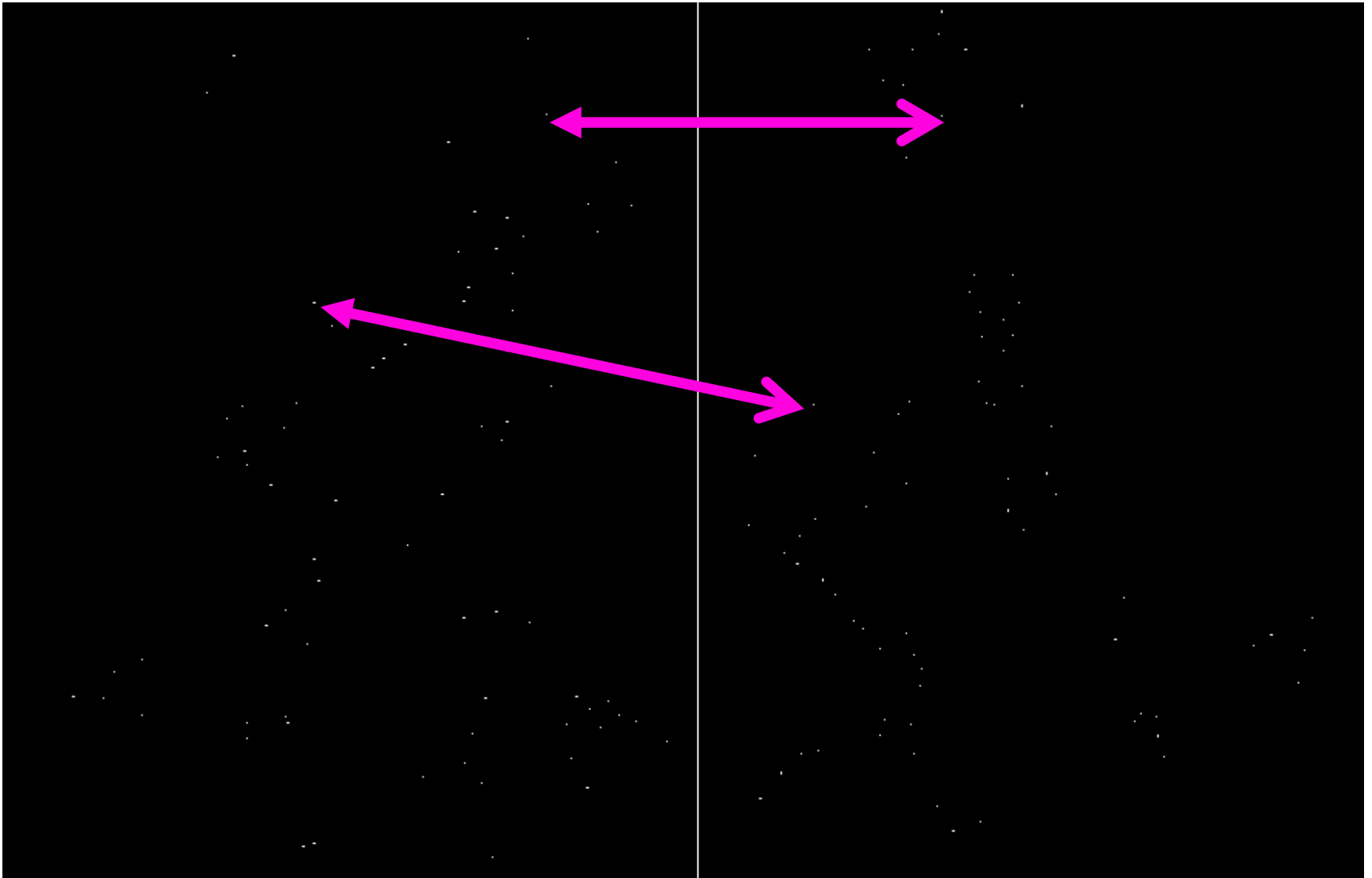
Harris Detector: Workflow

Find points with large corner response: $R > \text{threshold}$



Harris Detector: Workflow

Take only the points of local maxima of R



Harris Detector: Workflow

