CS5350: Machine Learning *Due 11 Nov 2008*

Project 3: Applied Learning Theory

Introduction

In project 2, we are looking at the applied side of learning theory. There are two components of this project: sides of learning theory we've considered:

1. Boosting (AdaBoost)
2. Reductions from multiclass to binary classification

There are two data sets: `5ng-test` for real testing (this is a five-class subset of the 20 newsgroups data) and `simpledata`, a synthetic and small data set.

Task 1: Boosting *(40%)*

In this task, we will implement AdaBoost. A shell for AdaBoost is in `Boost.m`. You will need to fill in the `BoostTrain` and `BoostPredict` functions. See the comments in `Boost.m` for a description of what each function should do and how it should store its outputs. Note that in order to do boosting properly, you need to have binary classification algorithms that can accept weighted data. I have provided an implmentation of decision trees that does so (in `DT.m`). The DT implementation has a maximum-depth parameter (as opposed to threshold) for pruning. This is to make it easier to train decision stumps.

Once you've implemented `Boost.m`, I suggest testing it on the `simpledata` data set using decision stumps as the base learner. To do this, first load the data:

```
>> trX = load('simpledataX');
>> trY = load('simpledataY');
```

Next, let's just train a simple decision stump and evaluate its performance on the training data:

```
>> model = DT('train', trX, trY, ones(size(trY)), 1);
>> mean(trY == DT('predict', model, trX))
ans = 0.82000
```

In the first line, we're building the decision tree. `trX` and `trY` are the training data points and labels. The `ones(...)` command is for the *importance weights* of the training points—they are all equally weighted. The final argument is the maximum depth. A stump has depth 1.

In the second line, we are computing the accuracy of the decision tree on the training data, by computing the mean of whether the predictions match the truth. In this case, we get 82% accuracy.

(If you retrain with maximum depth 2, you should get 86% accuracy.)

Now, we boost the model. In order to call `Boost`, we need to supply it with the base learner we want to use (in this case, `DT`), the training data, the number of rounds of boosting to perform, and any extra arguments to the base learner. In this case, to build stumps, we want to pass `1` as an additional argument. We then evaluate. Let's try boosting a few different number of iterations (one through 4). Note the alphas and errors that are created as part of the boosted model:

```
>> model = Boost('train', @DT, trX, trY, 1, 1)
model =
    models: {[1x1 struct]}
    alphas: {[0.7582]}
    errors: {[0.1800]}

>> mean(trY == Boost('predict', @DT, model, trX))
ans =
    0.8200

>> model = Boost('train', @DT, trX, trY, 2, 1)
model =
    models: {[1x1 struct]  [1x1 struct]}
    alphas: {[0.7582]  [0.5367]}
    errors: {[0.1800]  [0.2547]}

>> mean(trY == Boost('predict', @DT, model, trX))
ans =
    0.8200

>> model = Boost('train', @DT, trX, trY, 3, 1)
model =
    models: {[1x1 struct]  [1x1 struct]  [1x1 struct]}
    alphas: {[0.7582]  [0.5367]  [0.3186]}
    errors: {[0.1800]  [0.2547]  [0.3459]}

>> mean(trY == Boost('predict', @DT, model, trX))
ans =
    0.8600

>> model = Boost('train', @DT, trX, trY, 4, 1)
model =
    models: {[1x1 struct]  [1x1 struct]  [1x1 struct]  [1x1 struct]}
    alphas: {[0.7582]  [0.5367]  [0.3186]  [0.0625]}
    errors: {[0.1800]  [0.2547]  [0.3459]  [0.4688]}

>> mean(trY == Boost('predict', @DT, model, trX))
ans =
    0.8600
```

What we see is that after three iterations of boosting, performance has gone from 82% to 86%. If we boost more, it doesn't really help. We can try, for instance, ten iterations:

```
>> model = Boost('train', @DT, trX, trY, 10, 1)
model =
    models: {1x10 cell}
    alphas: {[0.7582]  [0.5367]  [0.3186]  [0.0625]  [0.0570]
             [0.0301]  [0.0060]  [0.0022]  [0.0018]  [5.3642e-04]}
    errors: {[0.1800]  [0.2547]  [0.3459]  [0.4688]  [0.4715]
             [0.4850]  [0.4970]  [0.4989]  [0.4991]  [0.4997]}
```

Task 2: All-versus-all Classification *(30%)*

The second thing we'll implement is all-versus-all classification. As a reference, and so we can compare results later, I've implemented *one-versus-all* classification in `OVA.m`. Your job is to implement `AVA.m`, which, as usual, is partially written for you. Since `simpledata` is just binary, we can't test OVA and AVA on it, so we'll have to use the `5ng` data set. Let's load the data and inspect it:

```
>> load 5ng
>> size(trX)
ans =
  2795  3530

>> size(teX)
ans =
  1873  3530

>> myunique(trY)
ans =
  1
  2
  3
  4
  5
```

(Note, `myunique` returns the unique values in a vector: here, we see that there are five unique classes.)

Warning: if you're on Octave you'll have to load `5ng-octave` instead. Moreover, the lack of support for sparse matrices will make this part slow.

For initial code-testing purposes, let's use a small subset of the data for training and testing. We can create these as follows:

```
>> trX1 = trX(1:20:end,1:10:end);
>> trY1 = trY(1:20:end);
>> trX2 = trX(2:20:end,1:10:end);
>> trY2 = trY(2:20:end);
```

We'll use the "1" data for training and the "2" data for testing. These should contain 140 examples each and 353 features.

We'll start by running `OVA`, which will work without any effort on your part. We'll use a decision tree as the base learner with a maximum depth of 3. As `OVA` runs, it will tell you which class it's building a classifier for. We'll then check training error and test error:

```
>> ova = OVA('train', @DT, trX1, trY1, 3);
1...2...3...4...5...
>> mean(trY1 == OVA('predict', @DT, ova, trX1))
ans = 0.50714
>> mean(trY2 == OVA('predict', @DT, ova, trX2))
ans = 0.27857
```

These accuracies are pretty crummy, but that's largely because we built very shallow DTs on a very small subset of the data.

Now, that's all well and good, so let's try your `AVA` code. We'll do basically the exact same thing:

```
>> ava = AVA('train', @DT, trX1, trY1, 3);
1...2...3...4...5...
>> mean(trY1 == AVA('predict', @DT, ava, trX1))
ans = 0.50714
>> mean(trY2 == AVA('predict', @DT, ava, trX2))
ans = 0.29286
```

So we see that on the test data, AVA seems to be doing slightly better. Note that since DT supports multiclass "naturally," we can also just run the DT:

```
>> model = DT('train', trX1, trY1, ones(size(trY1)), 3);
>> mean(trY1 == DT('predict', model, trX1))
ans = 0.40714
>> mean(trY2 == DT('predict', model, trX2))
ans = 0.27857
```

As we can see, the AVA actually *outperforms* the internal multiclass handling of DTs (though I wouldn't trust this too much—this is a really tiny, unrealistic dataset).

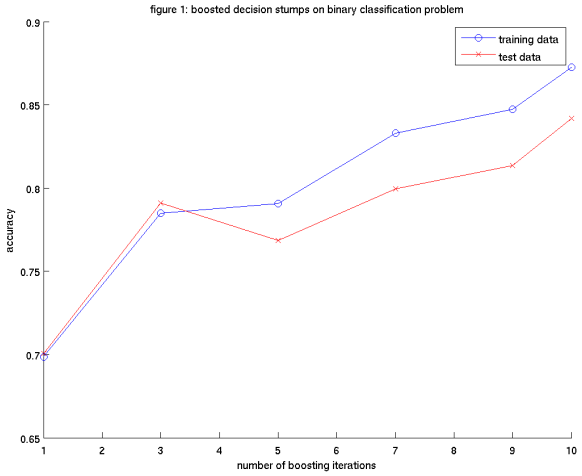
Putting it all together... *(30%)*

The `run.m` script is just like all other projects. When you execute `run.m`, it will attempt to load data from `results.mat` and will also attempt to detect if you're running Matlab or Octave (plotting commands vary slightly between the two, much to my chagrin). It will tell you what it's doing and if it's wrong, you might want to go in and just hard-code the correct responses. Next it will load data from `5ng` (or `5ng-octave`, if it thinks you're running Octave). It then runs the following set of experiments, each of which leads to a single plot. (If you rerun `run.m` later, with `result.mat` saved, it will still regenerate the plots, but will not run all the classifiers.) I've also listed the amount of time that it takes my solution to run for each part, so you have a sense. The total time was 32 minutes.

1. Boosted decision stumps on binary data, with variable number of boosting iterations. *(2 minutes)*
2. Boosting decision *trees* on binary data, with varying tree depth. *(5 minutes)*
3. Multiclass-to-binary reduction with DT, with learning curve. *(13 minutes)*
4. Multiclass-to-binary reduction with DT, with varying number of classes. *(14 minutes)*

Each figure is now labeled better and the questions below refer to each of the previous steps (each of which produces it's own figure).

F1: figure 1 is attached below:



(a) In this figure, we see training (blue) and test (red) accuracy curves for different amounts of boosting. Do you observe any overfitting or underfitting?

Answer: I can not see any overfitting from this single figure. With more iteration, the training accuracy rate goes up, and also the test accuracy goes up at same time. This is because decision stump is a simple model and it's hard to get overfitted.

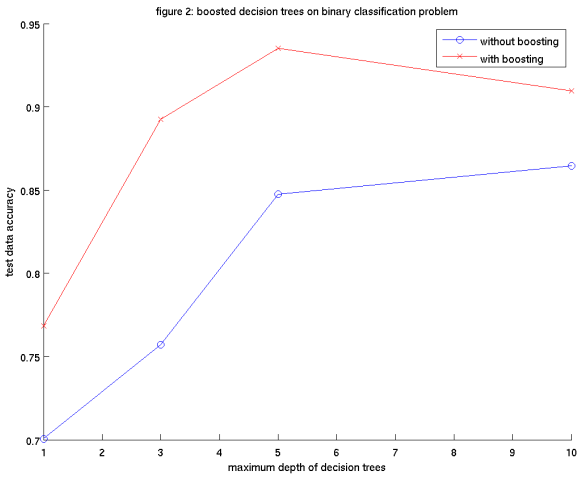
for underfitting, we'll suspect underfitting when both the training error and test error are large. In this figure, when iteration time is 10, we see both the training and test accuracy are above 80%, which is acceptable. Hence, we can not see any underfitting from this figure.

(b) How would you decide when to stop boosting?

One method is to see if ϵ is close enough to 0.5. if $\epsilon \approx 0.5$, it means $\alpha_t \approx 0$, and we dont change the distribution much. At this point, we can stop the boosting.

Another intuitive method is to see if the accuracy on training set is stable enough with increasing iteration time, though this does not mean the accuracy on test set has no room to improve. For decision stump, we see the training accuracy and test accuracy increase consistently with the boosting time, so if the training accuracy come to a constant, we can assume test accuracy also does.

F2: figure 2 is attached below:



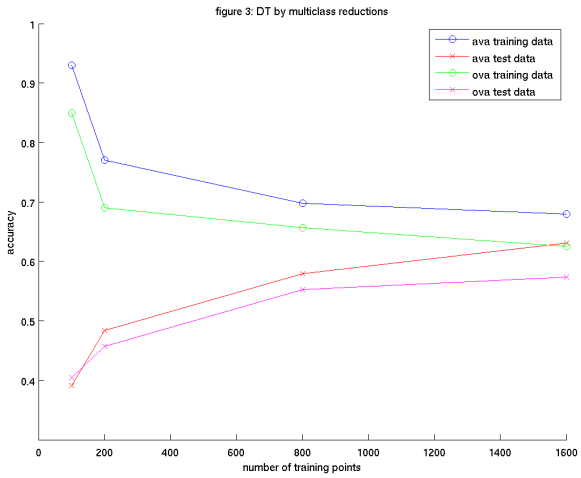
(a) Here we see the effect of boosting not just stumps (decision trees with depth 1), but also fuller decision trees. The curves are both for test accuracy. Where does boosting help the most and the least?

Answer: From figure 2, we can see when we boost five times, the test accuracy with boosing is much better than test accuracy without boosting. When boosting ten times, boosting seems doesnt help a lot.

(b) What do you think would happen if we let the the maximum depth get larger?

Answer: When we have deeper decision tree, both the decision tree without boosting and the tree with boosting will be more sensitive to the noise, and tends to be overfitting. However, as boosting focus on the 'mis-classified' samples and put more weight on them, it is even more sensitive to noise. So when the tree depth increases, I would expect the difference between boosting and non-boosting will be less, especially when we have more boosting time.

F3: figure 3 is attached below:



(a) Here, we see learning curves for multiclass reductions for perceptron, using both AVA (green, purple) and OVA (blue, red). The two top curves (blue, green) are training data; the two bottom (red, purple) are test data. Between OVA and AVA, which seems to work better. Why do you think this is?

Answer: AVA works better on both training data and test data, especialy when there is large data points. This is because when data set is large, some noise is introduced, For OVA, if one classifier misclassify the test data due to the noice in the training data, that will have large impact on the results. For example, if two binary classifiers classify test sample x as positive, we have to randomly choose one of them as the final results. And this add the error rate.

OVA is a combination of various classifiers. If one classifier misclassify the test sample, it may not necessarily change the final result if it only have small weight (i.e. the α).

(b) Discuss the effect that you see on the training curves, based on what we know about these two algorithms.

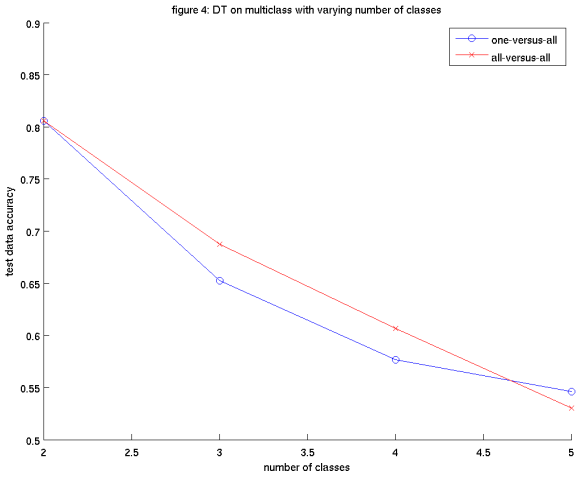
Answer: the first thing we noticed is when there are small size of training data, both OVA and AVA boosting have better results than the accuracy on the test data. This can be seen as a kind of overfitting, because boosting algorithm use the few data again and again in order to fit it. It adjust its hypothesis space specifically for those incorrectly classified, and at last, boosting can always get good claissificatoin accuracy. But that does not mean it has learned the correct 'concept'.

When the size of data set increase, the accuracy on training data decreases for both AVA and OVA. This is because it's harder for boosting to train *evey* training points correctly even after many times of boosting. Besides, there is more noise in large size data that is hard to fit.

When the size of data set increases, the accuracy on test set increase for both AVA and OVA. This is because by learning from large size of data, boosting learned a better hypothesis that close the 'concept', though it can not fit every training points. Large data always mean more information.

The more data set we have, the better performance that AVA has than OVA. This is OVA tends to learn the incorrect hypothesis (or classificatioin plane) from the noise in the large data set. AVA avoid this problem because its error is at most twice the average loss on the each binary classifiers.

F4: figure 4 is attached below:



(a) In this plot, we see OVA and AVA test accuracies on a varying number of classes (from 2 to 5). Note that we expect the accuracies to go down as the number of classes increases (since the problem is getting harder). However, does one reductions seem to fare better (relatively) as the number of classes increases? How does this relate to the theoretical analysis we had of these algorithms?

Answer: AVA performs better than OVA when there are less classes. When there are more classese, the AVA have $\binom{C}{2}$ binary classes, where C is the number of classes. This large amount of classes tends to overfit the data, because the data set size does not increase. The number of binary classes of OVA does not increase that much compared to AVA, so the test accuracy does not go down significantly than AVA.

What to hand in

Please hand in (a) your `results.mat` file and (b) a `writeup.pdf` containing the answers to all the questions in the previous section. Note that you do *not* have to discuss anything related to the "toy" data set.