Sample Space: sets whose elements are the outcomes we are interested in.

Discrete random variable: a function that maps a sample to a discrete number

\[ p(x) = p(X = x) \]

Cumulative distribution function

\[ F(a) = P(X \leq a) \]

### Bernoulli Distribution

\[ P_X(1) = P(X = 1) = p \]
\[ P_X(0) = P(X = 0) = 1 - p \]

\[ X \sim \text{Ber}(p) \]

\[ \binom{n}{k} = \frac{n!}{k! (n-k)!} \]

### Binomial Distribution

\[ P_X(k) = P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} \]

\[ \text{Bin}(n, p) \]
Geometric Distribution

Example: Student looks for interview at career fair booths.

At each booth:

\[ P(\text{off-campus interview invitation}) = p \]

\[ X: \text{the number of companies that student gets the first invitation.} \]

\[ P(X = k) = P(\text{No invitation at } k-1 \text{ booths}) \quad \cdot \quad P(\text{invitation at } k) \]

Definition:

\[ P(X = k) = (1-p)^{k-1} \cdot P_{\text{Geo}}(p) \]

Exer.: Show: \[ P(X > k) = (1-p)^k \]

Sampling Space: \[ \{S, FS, FFS, FFFS, \ldots \} \]

R.V.: \[ 1 2 3 4 \]

(mapping)
A series of visiting company as a single experiments

Each visit can be an event, but since we are interested in the visits needed for the 1st interview, we define experiments this way.

This is like the HW1 B/w card. We can define drawing a card as an experiment, but computation more difficult. So we define drawing it looking at top as a single experiment.
Exer 4.5

Throw a die until the sum exceeds 6.

\[ X: \text{# of throws}, \quad F(1), \ F(2), \ F(7) \]

Remember \( F(a) = P(X \leq a) \)

\[
F(1) = P(X \leq 1) = P(X = 1) = 0
\]

\[
F(2) = P(X \leq 2) = P(X = 2) = \frac{21}{6 \times 6}
\]

1: 6
2: 5, 6
3: 4, 5, 6
4: 3, 4, 5, 6
5: 2, 3, 4, 5, 6
6: 1, 2, 3, 4, 5, 6

\[
F(7) = P(X \leq 7) = 1 - P(X > 7) = 1
\]

21 ways
Exercise 4.6: Draw three times from \( \{1, 2, 3\} \)

\[
\bar{X} = \frac{x_1 + x_2 + x_3}{3}
\]

\[
\bar{X} = \frac{3}{3} = (1, 1, 1)
\]

\[
\bar{X} = \frac{4}{3} = (1, 1, 2); \ 3 \text{ ways}
\]

\[
\bar{X} = \frac{5}{3} = (1, 2, 2); \ 3 \text{ ways}
\]

\[
\bar{X} = \frac{6}{3} = 123; \ 6 \text{ ways}
\]

\[
\bar{X} = \frac{7}{3} = 223; \ 3 \text{ ways}
\]

\[
\bar{X} = \frac{8}{3} = 233; \ 3 \text{ ways}
\]

\[
\bar{X} = \frac{9}{3} = 333; \ 1 \text{ way}
\]

1. Two draws are exactly 1

\[
\Pr(A) = \frac{3+3}{3 \times 3 \times 3}
\]
Example (3). Geometric Distribution

Student visit career fair booth for interview

\[ P: \text{an invitation in a company booth.} \]

Well dressed: \( p = 0.8 \)

Badly dressed: \( p = 0.1 \)

1. PMF of \# of companies before getting an invitation

\[ p_x(k) = P(X = k) = (1-P)^{k-1} \cdot P \]

2. Well dressed stu get invitation in first 3 visits

\[ P(X \leq 3) = F(3) = P(X=1) + P(X=2) + P(X=3) \]

CDF disjoint

\[ P(X=1) = p = 0.8 \]

\[ P(X=2) = (1-p) \cdot p = 0.2 \times 0.8 = 0.16 \]

\[ P(X=3) = (1-p)^2 \cdot p = 0.2^2 \times 0.8 = 0.032 \]
\[ p(X \leq 3) = 0.8 + 0.16 + 0.032 = 0.992 \]

\[ p(X \geq 3) = p(X=4) + p(X=5) + \ldots \]
\[ = \sum_{k=4}^{\infty} p(X=k) \]
\[ = \sum_{k=4}^{\infty} p(1-p)^{k-4} \]

\[ = (1-p)^3 \cdot p \cdot \sum_{k'=0}^{\infty} (1-p)^k' \]

geometric sequence \[ a_n = ar^{n-1} \]
\[ a \quad ar \quad ar^2 \quad \ldots \]
\[ r = (1-p) \quad \sum_{k=0}^{n-1} ar^k = \frac{a(r^n)}{1-r} \]

b.c. \[ \sum_{k=0}^{n-1} ar^k = \frac{a(r^n)}{1-r} \]
\[ \sum_{k'=0}^{\infty} (1-p)^{k'} = \frac{1 - (1-p)^n}{1 - (1-p)} \Rightarrow \frac{1}{p} \]

\[ p(X > 3) = (1-p)^3 \cdot p \cdot \frac{1}{p} = (1-p)^3 \]
Simpler solution, since first 3 visits fail
\[ p(X > 3) = (1 - p)^3 \]

4) If a student got an interview invitation from his 4th visit, what is his probability of well dressed? (Assume well/badly addressed have equal prob.

→ a r.v. that map “well dressed”
to a discrete number.

\[ p(d=1 | X=4) = \frac{p(d=1) \cdot p(X=4 | d=1)}{p(X=4)} \]

\[ p(X=4) = p(d=1) \cdot p(X=4 | d=1) + p(d=0) \cdot p(X=4 | d=0) \]

\[ = 0.5 \times (1-0.8)^3 \times 0.8 + 0.5 \times (1-0.1)^3 \times 0.1 \]

\[ = 0.03965 \]

\[ p(d=1 | X=4) = \frac{0.0032}{0.03965} \approx 0.08 \]

If a stu got interview at 4th visit, s/he must be dressed badly.
\[ p(\text{odd}) = p(x=1) + p(x=3) + \ldots \]
\[ = (1-p)p + (1-p)^2p + \ldots \]

\[ p(\text{even}) = p(x=2) + p(x=4) + \ldots \]
\[ = (1-p)p + (1-p)^3p + \ldots \]
\[ = (1-p) \left[p + (1-p)^2p + \ldots \right] \]
\[ = (1-p) \cdot p(\text{odd}) < p(\text{odd}) \]