GASP: Gradient Aware Shortest Path Algorithm for Boundary-Confined Visualization of 3D Reeb Graphs

Sefat E Rahman, Tushar Athawale, Paul Rosen
University of Utah, Oak Ridge National Lab

Introduction

What is a Reeb graph?

A Reeb graph is a mathematical concept used to analyze and understand the topology (shape and structure) of a manifold or scalar field, particularly in multidimensional data.

Why are Reeb graphs important?

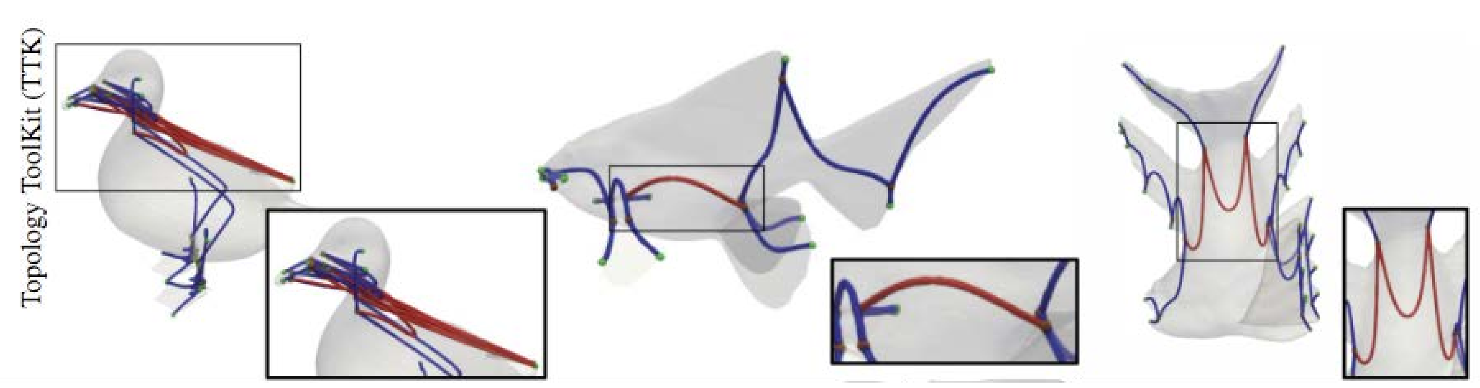
Reeb graphs play a crucial role in the structural analysis and visualization of scalar field data on manifolds, applicable in diverse fields from medical imaging to geography.

Motivation

Despite their utility, existing visualization techniques often misrepresent the data through arcs that extend beyond the model boundary, follow unnecessarily long paths, or inadequately align with the data's gradient flow. The paper introduces a new algorithm, GASP, aimed at improving the visualization of 3D Reeb graphs. By focusing on boundary-constrained, shortest-path, and gradient-aware visualizations, GASP addresses shortcomings in existing methodologies like the Topology ToolKit (TTK), particularly in accurately representing the underlying height function of models

Challenges Addressed:

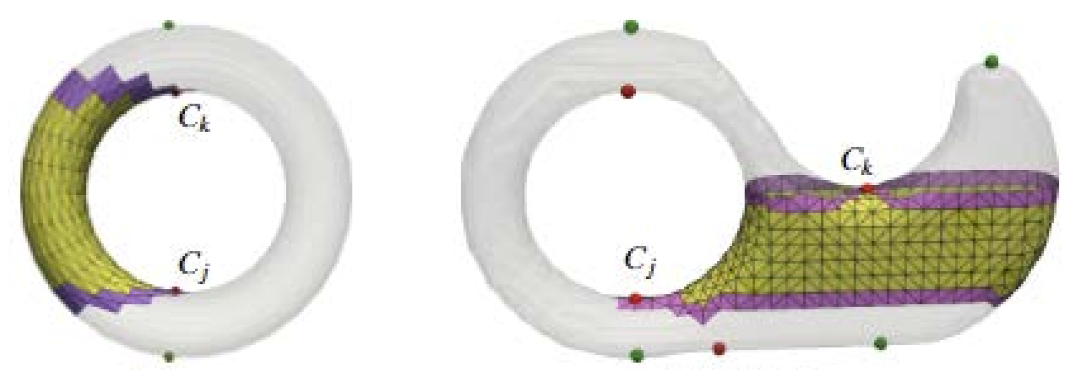
- **Arcs Outside the Model:** Traditional approaches may generate Reeb graph arcs that extend beyond the model's boundary, leading to inaccurate representations.
- **Unnecessarily Long Arcs:** Existing methodologies can result in Reeb graph edges that are longer than needed due to smoothing processes that disregard the shortest possible path.
- **Misalignment with Gradient:** Previous methods sometimes fail to align Reeb graph arcs with the function's gradient, compromising the visualization's fidelity to the actual data structure.



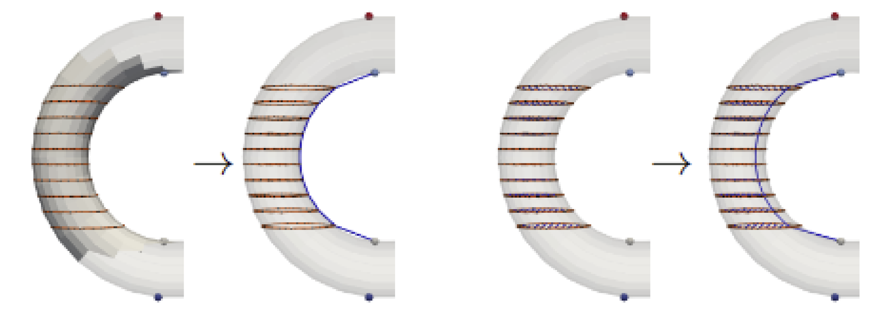
Proposed Solution: GASP Algorithm

GASP employs a three-step process:

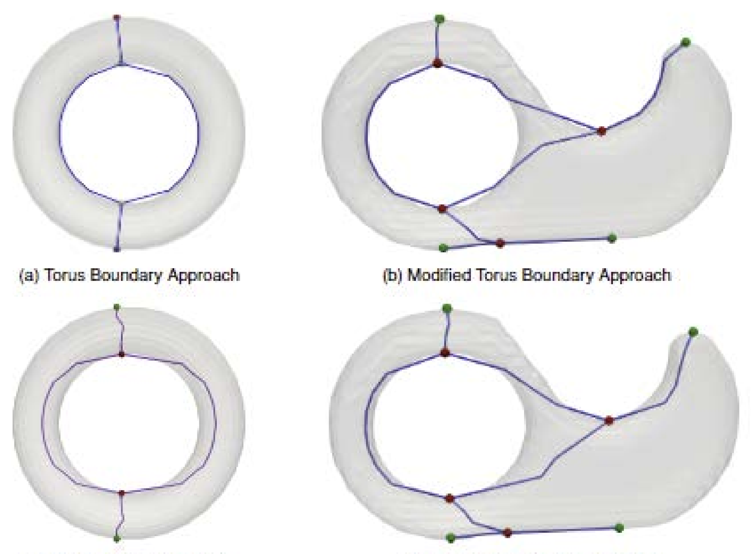
- **Decomposition:** The model is decomposed into topological cylinders associated with Reeb graph edges.



- **Reeb Graph Arc Visualization**: For each cylinder, GASP calculates the shortest path that conforms to the model's boundary and aligns with the gradient of the height function.



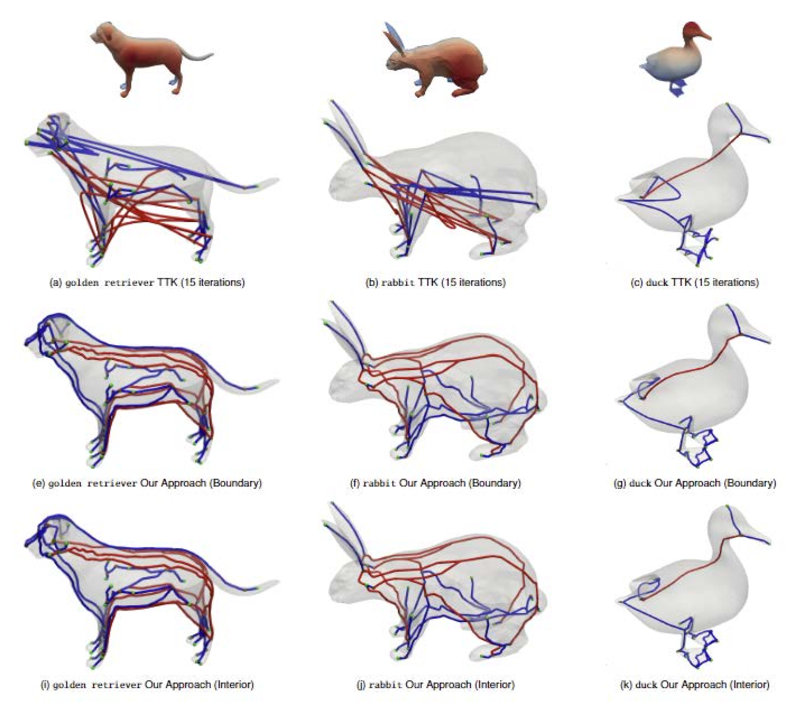
- **Final Assembly:** The individual arcs are then assembled to form the final Reeb graph visualization.



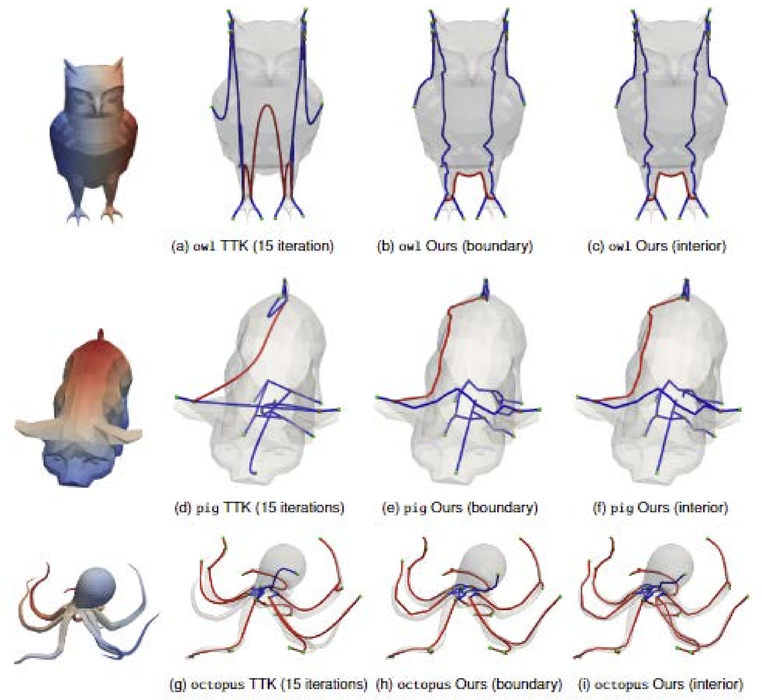
This approach ensures that Reeb graph arcs remain within or on the surface of the object, take the shortest route between critical points while being constrained by the boundary, and better align with the elevation function's gradient.

Evaluation

- **Boundary Constraint**



- **Shortest Path Arcs**



- **Gradient Aware**