Department of Mathematics, University of Utah

Introduction to Mathematical Finance
MATH 5760/6890 – Section 001 – Fall 2024
Homework 1
Simple valuations

Due: Friday, Aug 30, 2024

Submit your homework assignment on Canvas via Gradescope.

1.) (Loan valuation) Suppose the (annual) interest rate for a loan is currently 3% for a loan term of 3 years, and the interest will compound monthly. You estimate that you will be able to afford a maximum monthly payment of $300 over the loan term. What is the maximum loan amount you can take out and still afford the monthly payments?

2.) (Compound interest) You decide that starting today you will deposit $300 per month in a money market account with an annual interest rate of 5% compounded daily (ordinary interest). Deposits are made in lump sums of $300 on the first day of each month (30-day period), and today is the first day of the month. After 3 years time (36 deposits total), how much money will you have in the account? What is the minimum monthly deposit required so that the account will be worth $15,000 after 3 years?

3.) (Simple options) A *call option* is a contract between a buyer and seller that entitles the buyer the right (but not the requirement) to purchase a fixed amount of stock shares at a specified, "strike" price: The buyer agrees to pay the seller a premium today, which gives the buyer the right to force the seller to sell the shares at the strike price anytime between today and the expiry term of the option. (Hence, the sale completes based on today's agreed upon strike price, not based on market price on the day of the sale.) A "European" option is one where the buyer is permitted to exercise the option only at expiry (not before). Assume rational acting: the buyer exercises only if it is a rational choice (i.e,. only if the market price at expiry is greater than the strike price).

(a) In entering to a European call option contract, briefly explain what the buyer hopes will happen to the future stock price (and why), and what the seller hopes will happen (and why).

(b) Let ABC be a stock selling on the market today for $5 per share. Suppose a European call option premium is $100 for 100 shares of ABC at strike price $6 per share.[1] Plot the buyer's profit as a function of ABC's expiry share price.

(c) Consider a *put option*, in which the seller pays a premium to gain the right (but not the requirement) to sell stock to the buyer at the strike price. With the same fees, parameters, and rational acting as in the previous part, plot the seller's profit on a European put option as a function of ABC's expiry share price.

4.) (Arbitrage) Consider the options setup above (same parameters and dollar amounts for both call and put options). Suppose you have the ability to invest (take long positions)

[1] Options are commonly sold in units of 100 shares.

in 3 securities: European call options for ABC, European put options for ABC, and a money market account with an annual rate of 4% compounded daily (ordinary interest). All options have an expiry of 1 year. Assume you have \$10,000 available to invest, and that you will invest all of it in a combination of these 3 securities. You may only purchase whole units of options, no fractional purchases are allowed; the money market investment has no limitations on the investment amount.

(a) Identify all actions that result in arbitrage at expiry of the options.

(b) From the collection of actions identified above, describe what knowledge about ABC would make one action more favorable than another.