Department of Mathematics, University of Utah

Introduction to Mathematical Finance
MATH 5760/6890 – Section 001 – Fall 2024
Homework 2
More valuations

Due: Friday, Sept 6, 2024

Submit your homework assignment on Canvas via Gradescope.

1.) (Bond valuation) Bonds are priced based on today's present value of the instrument; the "face value" of a bond is the amount paid to the bearer at expiry of the bond's term ("maturity"). In the simplest setting, the price is based on the sum of two things: (i) the present value of the face value of the bond (which is received at maturity), (ii) the present value of an annuity ("coupon payment") paid to the holder at regular intervals; the payment per period equals the face value times the "coupon/interest rate" (prorated from a quoted annual rate). The "yield to maturity" is an assumed (annual) interest rate on which the present value is discounted from future value.

(a) Determine the price of the following bond: A bond with a face value of \$1000 with a maturity term of 2 years and a coupon rate of 4%. The coupon payments are made semiannually (i.e., twice a year at \$20 per payment) until the bond matures. Throughout, assume a discount rate ("yield to maturity") of 3%.

(b) "Par" refers to a bond face value. Is the bond in part (a) priced below, at, or above par?

(c) (**6890 students only**) Prove in general that if the yield to maturity equals the coupon rate, then the present value of the bond is exactly par.

2.) (Forward price) Consider entering a into forward contract with a maturity term of 1 year as a buyer for a single share of a company XYZ. Suppose that today's share price is \$100 per share, and that the company has committed to paying \$1, \$2, \$3, and \$5 in dividends at the end of quarters 1, 2, 3, and 4, respectively (a quarter is three months). Suppose in this market there is also a risk-free security with an annual interest rate of 4% (e.g., a savings account rate), compounded continuously. Assume you are a *rational pricer*, meaning that you value securities based on an assumption that there is no arbitrage in the market. Determine a fair forward price for this contract. (Recall that dividends implicitly decrease the value of a corporation.)

3.) (Long and short payoff diagrams) In this problem assume the underlier is a single share of a stock. Plot 6 payoff diagrams: for both long and short positions for a forward contract, a European put option, and a European call option. For concreteness, suppose \$10 is the strike price for both of the options and also the forward price for the forward contract. (Payoff diagrams plot gross revenue at maturity/expiration as a function of the underlier price, ignoring premium.)

4.) (Put-call parity) Consider a forward contract with forward price F. Suppose S is the actual market price of the underlying asset at maturity, with maturity being T years.

(a) Using a rational pricing assumption with continuously compounded annual interest rate r, compute the *value* of this forward contract today, i.e., a reasonable premium today for entering into a long position in this contract.

(b) Using the results from the previous problem, use long and/or short positions in European call and/or put options to construct a payoff that matches a single long forward contract.

(c) Let P be the *value* of a put option today (e.g., the premium), and let C be the value of a call option. Derive a relation between the values of forward contract, put, and call options. (This relation is called *put-call parity*, and requires us to assume that the underliers are all the same, the strike price and forward price are the same, and that the expiry for all the contracts is the same.)