Department of Mathematics, University of Utah

Introduction to Mathematical Finance
MATH 5760/6890 – Section 001 – Fall 2024
Homework 7
The Binomial Pricing Model

Due: Friday, Oct 25, 2024

Submit your homework assignment on Canvas via Gradescope.

1.) Consider an $n = 100$-period Binomial Pricing Model with $(p, u, d) = (0.35, 1.2, 0.9)$ and an initial value of $S_0 = 100$.

(a) What is the maximal value of S_{100} under this model? The minimal value?

(b) Compute the probability that $S_{100} \geq 100$.

2.) Let $X \sim \text{Bernoulli}(p)$, and let $\{X_i\}_{i=1}^n$ be n iid copies of X. Let $Y = \sum_{i=1}^n X_i \sim \text{Binomial}(n, p)$. Throughout this problem, let $a > 0$ be a deterministic constant.

(a) Compute $\mathbb{E}a^X$ with $a > 0$ a deterministic constant.

(b) If V and W are two independent random variables, then $\mathbb{E}(VW) = (\mathbb{E}V)(\mathbb{E}W)$. Use this to compute $\mathbb{E}a^Y$.

(c) Compute the variance of a^Y.

(d) Apply these facts to the Binomial Pricing Model with parameters (p, u, d): with $S_n = S_0 e^L$, where $L = \sum_{i=1}^n L_i = \sum_{i=1}^n \log G_i$ is the log-return, show that,

$$\mathbb{E}\frac{S_n}{S_0} = (pu + (1-p)d)^n,$$
$$\text{Var}\frac{S_n}{S_0} = \left(pu^2 + (1-p)d^2\right)^n - (pu + (1-p)d)^{2n}$$