

Submit your homework assignment on Canvas via Gradescope.

- 1.) (Petters & Dong, Problem 5.5) For an n -period binomial tree, let N_U be the number of security price upticks from time t_0 to t_n . Explain why N_U is a binomial random variable. What are its expected value and variance if the tree has 40 steps and the uptick probability is 60%?

- 2.) Consider an n -period Binomial Pricing Model for an asset over the time interval $t \in [0, T]$ with $d = 1/u < 1$. (This is a special type of recombination condition.)
 - (a) Show that the expected value of the gross return, $\mathbb{E} \frac{S_n}{S_0}$, is given by $(pu + (1-p)/u)^n$. (You may use results from previous homework assignments if desired.)
 - (b) Suppose that from historical data we compute a T -time expected return rate r for the asset:

$$\mathbb{E}S_n = S_0(1 + r),$$

where r is a deterministic, positive constant $r > 0$. Show that in order for S_n under the given binomial pricing model to achieve an expected gross return rate $(1 + r)$, then u must satisfy,

$$u = \frac{1}{2p} \left(e^\mu \pm \sqrt{e^{2\mu} - 4p(1-p)} \right),$$

where $\mu = \frac{\log(1+r)}{n}$.

- (c) Show that $e^{2\mu} > 4p(1-p)$, and hence there are always two real values of u above.
- (d) Show that if we choose the minus option in formula with \pm above, then $u < 1$, and hence the only valid choice is the plus option.