Introduction to Mathematical Finance
MATH 5760/6890 – Section 001 – Fall 2024
Project 1
Portfolio construction

Due: Friday, Oct 18, 2024

Submit your assignment on Canvas via Gradescope.

In this project, you will construct a Markowitz portfolio and apply capital market theory to real data.

a. Identify <u>at least</u> 6 stocks/tickers that you wish to build into a portfolio. These can be stocks for publicly traded companies, or can themselves be hedge/mutual funds; however, they should be risky securities, i.e., they should have the potential to decrease in value.

b. Research and compile first- and second-order statistics for the above securities. You should compile the statistics necessary to formulate a Markowitz portfolio optimization.

c. Perform a Markowitz portfolio analysis, which includes (i) identifying the risk-optimal portfolio as a function of an investor's target expected return, (ii) plotting of the Markowitz efficient frontier, (iii) a computation of the mean and variance of the global variance-minimizing portfolio.

d. Identify a single risk-free security and its corresponding return (in the same time units as the risky securities). This risk-free security could be a government bond, or something as simple as a savings account or money market account interest rate that you could currently gain access to.

e. Using captial market theory, with the risk-free rate corresponding to the return of the risk-free security in the previous part, identify the market portfolio and the capital market efficient frontier.

Write a report-style submission for the above exercises. You need not identify your "answer" to each part; instead, write as if you were constructing a report for an investor or a potential client.

In order to gather statistics for the securities you identify, you can use analysis from historical data available on the internet. See., e.g., slide D11-S03 from the lecture slides for examples of websites that provide such data. When compiling statistics, note that:

- For first-order statistics, a "rate" or "average return" is what you are looking for. A gold standard is the *compound annual growth rate* (CAGR). Whichever you compute and collect, provide any information to identify roughly what historical time period the data used for analysis came from, and also what the time unit for the reported rate is. (E.g., annual, monthly, ...) Finally, if possible report whether or not the rates are inflation-adjusted.

- For second-order statistics, you'll often see standard deviations reported for single assets. However, covariances between assets will typically be reported as (Pearson) correlations. If X and Y are two random variables, their Pearson correlation ρ is defined as $\rho = \mathrm{Cov}(X, Y)/(\sigma_X \sigma_Y)$, where σ_X and σ_Y are the standard deviations of X and

Y, respectively. Thus, standard deviations and correlations are enough to determine covariances.