Department of Mathematics, University of Utah

Analysis of Numerical Methods I

MATH 6610 – Section 001 – Fall 2025

Homework 9

QR with shifts

Due Wednesday, November 5, 2025

Submission instructions:
Submit your assignment on gradescope.

Problem assignment:

1. Let $\boldsymbol{A} \in \mathbb{C}^{n \times n}$, and let $\boldsymbol{x}_0 \in \mathbb{C}^n$. State and prove a theorem, requiring appropriate assumptions, ensuring that (a properly normalized version of) $\boldsymbol{A}^k \boldsymbol{x}_0$ for large k converges to an eigenvector of $\boldsymbol{A}$.

2. Implement two algorithms for computing the full spectrum of a matrix $\boldsymbol{A} \in \mathbb{C}^{n \times n}$:

- Rayleigh iteration for normal matrices, in conjunction with an appropriate deflation procedure (such as Hotelling deflation).
- The QR algorithm with shifts for arbitrary matrices (e.g., with Rayleigh shifts)

In both algorithms you may lean on other/existing software to perform linear solves and/or to compute QR decompositions. Ensure that your algorithms work (i.e., correctly compute the spectrum) for matrices up to at least size 100. In your solution for this problem, provide:

- A brief but precise description of what each algorithm does. (E.g., pseudocode is fine, but a narrative description of the procedure is required.)
- Numerical results showing error (say error in the computed spectrum) as a function of matrix size. You may use an available numerical routine in the software of your choice to compute oracle/"exact" spectral information.
- Numerical results showing computational time required as a function of matrix size.
- Numerical results summarizing the number of iterations as a function of matrix size. (For Rayleigh iteration, the iterations performed on deflated matrices do count.)