Ex. Compute $\lim_{x \to -1} \frac{2+\sqrt{x+1}}{\sqrt{x+1}}$

DNE: $\sqrt{x+1}$ doesn't have a domain around $x=-1$

Ex. $\lim_{x \to 0} \frac{\sin(3x)+4x}{x \sec x} \underset{x=0}{\overset{?}{=}} \frac{0+0}{0} = \frac{0}{0}$ ☹

$= \lim_{x \to 0} \frac{\sin(3x)}{x \sec x} + \underbrace{\lim_{x \to 0} \frac{4x}{x \sec x}}_{x=0 \Rightarrow 4/1 = 4}$

$= \underbrace{\left(\lim_{x \to 0} \frac{1}{\sec x}\right)}_{1} \left(\lim_{x \to 0} \frac{\sin 3x}{x}\right) + 4$

$= 1 \cdot \lim_{x \to 0} 3 \frac{\sin 3x}{3x} + 4 = 3 \lim_{3x \to 0} \frac{\sin 3x}{3x} + 4$

$= 3 \lim_{y \to 0} \frac{\sin y}{y} + 4 = 3 \cdot 1 + 4 = 7$

2.20 : $g(x) = 1/\sqrt{3x}$

$g'(x) = ?$

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{\sqrt{3(x+h)}} - \frac{1}{\sqrt{3x}}}{h} \cdot \frac{\frac{1}{\sqrt{3(x+h)}} + \frac{1}{\sqrt{3x}}}{\frac{1}{\sqrt{3(x+h)}} + \frac{1}{\sqrt{3x}}}$$

$$= \lim_{h \to 0} \frac{\frac{1}{3(x+h)} - \frac{1}{3x}}{h\left[\frac{1}{\sqrt{3(x+h)}} + \frac{1}{\sqrt{3x}}\right]} = \text{simplify numerator}$$

...

3. (20 points)
For this problem, define $f(x)$ as follows:

$$f(x) = \frac{|x-1|}{x^2-1}$$

See the guidance from question 2 for acceptable values of a limit.

(i) (10 pts) Determine the values of c for which $\lim_{x\to c} f(x)$ does <u>not</u> exist. For these values, compute the one-sided limits $\lim_{x\to c^+} f(x)$ and $\lim_{x\to c^-} f(x)$.

"Problem": divide by 0: $x^2-1=0 \Rightarrow \underline{x=\pm 1}$

$\underline{x=+1}$ $\quad \lim_{x\to 1} \frac{|x-1|}{x^2-1} \quad \left(= \frac{0}{0}\right)$ $\qquad |-3| = -(-3) = 3$

compute $\lim_{x\to 1^+} \frac{|x-1|}{x^2-1} = \lim_{x\to 1^+} \frac{x-1}{x^2-1} = \lim_{x\to 1^+} \frac{1}{x+1} = \frac{1}{2}$

$\lim_{x\to 1^-} \frac{|x-1|}{x^2-1} = \lim_{x\to 1^-} \frac{-(x-1)}{x^2-1} = -\frac{1}{2}$

$\Rightarrow \lim_{x\to 1} f(x)$ doesn't exist, $\lim_{x\to 1^+} f(x) = \frac{1}{2}$

$\lim_{x\to 1^-} f(x) = -1/2$

$\underline{x=-1}$ $\quad \lim_{x\to -1} \frac{|x-1|}{x^2-1} = \lim_{x\to -1} \frac{|x-1|}{(x+1)(x-1)}$

$\lim_{x\to -1} \frac{|x-1|}{(x-1)} \cdot \lim_{x\to -1} \frac{1}{x+1}$ $\quad = -1 \cdot \lim_{x\to -1} \frac{1}{x+1}$

$\lim_{x\to -1^+} \frac{1}{x+1} = \frac{1}{\varepsilon}$ for ε very small, positive

$\Rightarrow \lim_{x\to 1^+} \frac{1}{x+1} = +\infty$

$\lim_{x\to -1^-} \frac{1}{x+1} = \frac{-1}{\varepsilon}$ for ε very small, positive

$= -\infty$

$$\left.\begin{array}{l} \lim_{x\to -1^+} f(x) = -\infty \\ \lim_{x\to -1^-} f(x) = +\infty \end{array}\right\} \Rightarrow \lim_{x\to -1} f(x)$$

DNE

$$f(x) = \frac{|x-1|}{x^2-1}$$

(ii) (3 pts) Determine the value(s) of c for which f is discontinuous at $x = c$.

Continuity @ $x=c$ means $\lim_{x \to c} f(x) = f(c)$

f is discontinuous @ $x = \pm 1$, e.g. because $f(\pm 1)$ doesn't exist.

(iii) (7 pts) For the value(s) of c identified in ~~(iii)~~ (ii), identify which value(s) are removable discontinuities and which are non-removable discontuities.

Removable discont: $\lim_{x \to c} f(x)$ exists, but is not $f(c)$.

$\lim_{x \to \pm 1} f(x)$ DNE, so $x = \pm 1$ are non-removable discontinuities