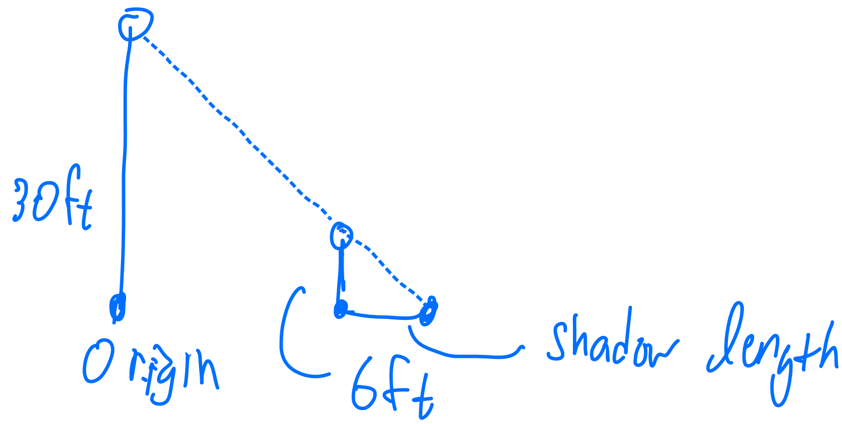
2.8.17 Chris is 6 feet tall, walking away from a 30 ft street light pole at a rate of 2 ft/sec.

(a) How fast is his shadow increasing in length when he's 24 ft from the pole? 30 ft?

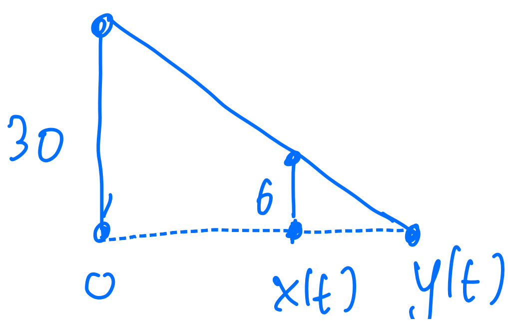
(b) How fast is the tip of his shadow moving?

(c) To follow the tip of his shadow w/ his eyes, at what angular rate must he be lifting his eyes when the shadow is 6 ft long?



let $x(t)$ be Chris's position

$$\frac{dx}{dt} = 2 \text{ ft/sec.}$$

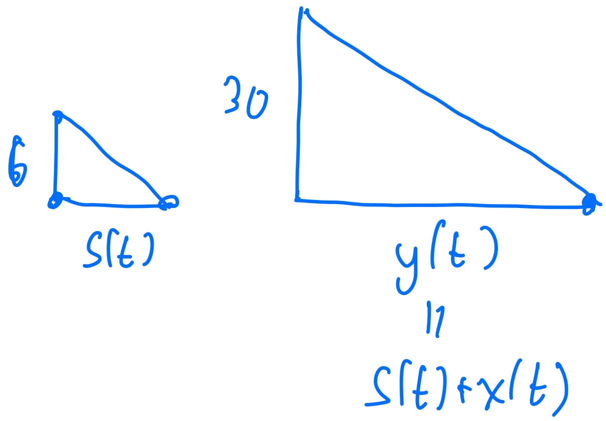


$y(t)$: position of shadow

$s(t)$: length of shadow: $s(t) = y(t) - x(t)$

(a) At time when $x(t) = 24$, what is ds/dt?

Similar triangles:



$$\frac{30}{6} = \frac{s(t)+x(t)}{s(t)}$$

$$5\,s(t) = s(t) + x(t) \rightarrow 4s(t) = x(t)$$

$$4\,s'(t) = x'(t) = \frac{dx}{dt}$$

$$s'(t) = \frac{1}{4}\,x'(t) = \frac{2}{4} = \frac{1}{2} \ \text{ft/sec.}$$

1. (40 points total)

Multiple Choice. Record your final answers here: circle or mark with an X your alphabetic answers for each of parts (i) - (iv).

(i)	A	B	C	D	E
(ii)	A	B	C	D	E
(iii)	A	B	C	D	E
(iv)	A	B	C	D	E

(i) (10 pts) Suppose $f(1) = 2$, $g(1) = 0$, $f'(1) = 0$, and $g'(1) = 3$. Compute $h'(1)$, where $h(x) = f(x)g(x)$.

A. $h'(1) = 3$

B. $h'(1) = 2$

C. $h'(1) = 0$

D. $h'(1) = 5$

E. $h'(1) = -1$

$h'(x) = f'(x)g(x) + f(x)g'(x)$

$h'(1) = f'(1)g(1) + f(1)g'(1)$

$= 0 \cdot 0 + 2 \cdot 3 = 6$

(ii) (10 pts) Express the derivative $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{F(x)}\right)$ in terms of the function $F(x)$.

A. $\frac{1}{F'(x)}$

C. $\frac{-F'(x)}{F^2(x)}$

B. $\frac{-1}{F^2(x)}$

D. 0

E. $F'(x)$

2. (20 points)

(i) (4 pts) Compute $y'(x)$ if $y(x) = \sin(x^3)$

(ii) (4 pts) Compute $y'(x)$ if $y(x) = \frac{x^2}{\cos x}$

(iii) (8 pts) If $y \sin x = \cos(xy)$, compute $y'(x)$ as a function of x and y.

$\big\downarrow d/dx$

$$y'\sin x + y\cos x = -\sin(xy)\cdot[1\cdot y + xy']$$

$$y'[\sin x + x\sin(xy)] = -y\sin(xy) - y\cos x$$

$$y'(x) = \frac{-y\sin(xy) - y\cos x}{\sin x + x\sin(xy)}$$

(iv) (4 pts) Compute the equation of the tangent line to the graph of $y(x)$ in the previous part (iii) at the point $(x, y) = \left(\pi, \frac{1}{2}\right)$.

$y'(x)$ @ $(x, y) = (\pi, 1/2)$

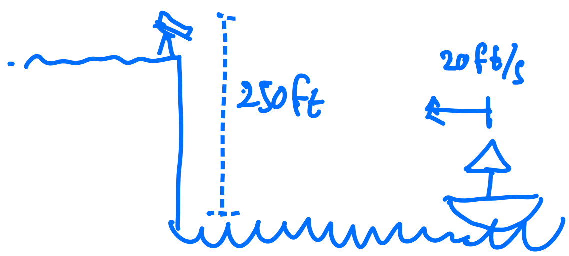
$$y' = \frac{-1/2\,\cancel{\sin(\pi/2)}^{1} - \frac{1}{2}\cancel{\cos(\pi)}^{-1}}{\cancel{\sin\pi}^{0} + \pi\,\cancel{\sin(\pi/2)}^{1}} = \frac{-\frac{1}{2}(1) - \frac{1}{2}(-1)}{0 + 1\cdot\pi}$$

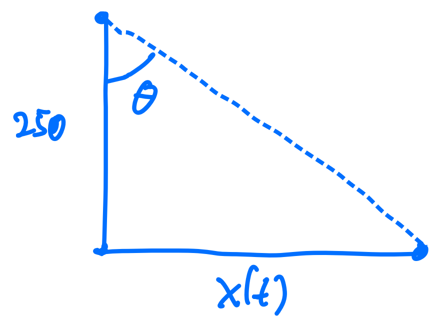
$$= 0 \text{ (slope)}$$

$$y - y_0 = m(x - x_0)$$

$$y - \tfrac{1}{2} = 0\cdot(x - \pi) \Longrightarrow y = 1/2$$

3. (20 points)
A woman standing on a cliff is watching a motorboat through a telescope as the boat approaches the shoreline directly below her. If the telescope is 250 feet above the water level and if the boat is approaching at 20 feet per second, at what rate is the angle of the telescope changing when the boat is 250 feet from the shore?





$x'(t) = -20$ ft/sec

$\frac{d\theta}{dt}$ when $x(t) = 250$?

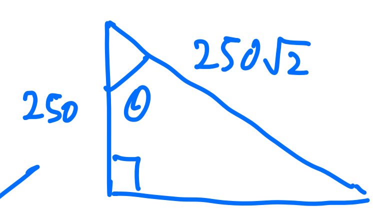
$$\tan\theta(t) = \frac{x(t)}{250}$$

quotient rule

$$[\sec^2\theta(t)]\cdot\theta'(t) = \frac{1}{250}x'(t)$$

$$\theta'(t) = \frac{x'(t)\cdot\cos^2\theta(t)}{250}$$

when $x(t) = 250$:



$\tan\theta = 250/250 = 1$

$\theta = \pi/4$

$\cos\theta = \frac{250}{250\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\cos^2\theta = \frac{1}{2}$

$\cos^2\theta(t) = \cos^2(\pi/4)$

$= \left(\sqrt{2}/2\right)^2 = 1/2$

$$\theta'(t) = \frac{x'(t)\cos^2\theta(t)}{250} = \frac{-20\cdot\frac{1}{2}}{250} = -1/25 \text{ rad/sec}$$

(This page is intentionally blank)