Practice Midterm Exam 3 **Name:** ______________________________
MATH1210, Section 023 April 4, 2025

This test is:

- closed-book
- closed-notes
- no-calculator
- 50 minutes

Indicate your answers clearly, and show your work.

For question 1, multiple choice: answers are graded purely based on final answers.

For questions 2-4, free-response: Partial credit will be awarded based on work shown. Full credit will not be awarded without some work shown.

The multiple choice question is worth 40 points. Each free response question is worth 20 points. (100 points total)

<u>Pages are two-sided. The first question begins on the back of this page!</u>

1. (40 points total)

Multiple Choice. Record your final answers here: circle or mark with an X your alphabetic answers for each of parts (i) - (iv).

(i)	A	B	C	D	E
(ii)	A	B	C	D	E
(iii)	A	B	C	D	E
(iv)	A	B	C	D	E

(i) (10 pts) What is the best description of what the Second Derivative Test is used for?

A. The value of the second derivative determines when a function is increasing or decreasing.

B. If a function can be differentiated twice, then it's an increasing function.

C. If the second derivative is the zero function, then the function is continuous.

D. When the second derivative vanishes, then the function has a local maximum.

E. The sign of the second derivative at a stationary point determines if it's a local maximum or minimum.

(ii) (10 pts) What is the iterative formula for Newton's method for solving the equation $f(x) = 0$ numerically?

A. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

C. $x_{n+1} = x_n - f'(x_n)$

B. $x_{n+1} = x_n$

D. $x_{n+1} + x_n = f(x_n) - f'(x_n)$

E. $\frac{x_{n+1}}{x_n} = f'(x_n)$

(iii) (10 pts) Suppose $F'(x) = f(x)$. Which of the following is always true?

A. $F(x)$ is an antiderivative of $f(x)$.

B. $f(x)$ is an antiderivative of $F(x)$.

C. $F(x)$ is the derivative of $f(x)$.

D. $F(x)$ and $f(x)$ have the same local maxima.

E. $F(x)$ is not a differentiable function.

$$\frac{d}{dx} F(x) = f(x)$$

(iv) (10 pts) Which of the following correctly interprets the definite integral $\int_a^b f(x)\,\mathrm{d}x$?

A. It's the signed area between the graph of $f(x)$ and the x axis on $[a, b]$.

B. It's the average slope of the tangent line to $f(x)$ on $[a, b]$.

C. It's the maximum value of $f(x)$ on $[a, b]$.

D. It's the minimum value of $f(x)$ on $[a, b]$.

E. It's the minimum slope of the tangent line to $f(x)$ on $[a, b]$.

2. (20 points)

(i) (7 pts) Find the general antiderivative of $f(x) = x^{100} + x^{99}$.

(ii) (7 pts) Find the general antiderivative of $f(x) = \frac{x^6 - x}{x^3}$.

(iii) (6 pts) Find the general antiderivative of $f(x) = x^2(x^3+4)^{14} - \sin x$.

$$\int f(x)\,dx = \int \left(x^2(x^3+4)^{14} - \sin x\right) dx$$

$$= \int x^2(x^3+4)^{14}\,dx - \int \sin x\,dx$$

$$= \int x^2(x^3+4)^{14}\,dx - (-\cos x + C)$$

Aside: $\frac{d}{dx}(x^3+4)^{15} = 15(x^3+4)^{14}\cdot 3x^2$
$= 45x^2(x^3+4)^{14}$

$$= \frac{1}{45}\int 45x^2(x^3+4)^{14}\,dx + \cos x - C$$

$$= \frac{1}{45}\left((x^3+4)^{15} + \tilde{C}\right) + \cos x - C$$

$$= \frac{1}{45}(x^3+4)^{15} + \cos x + \tilde{C}/45 - C$$

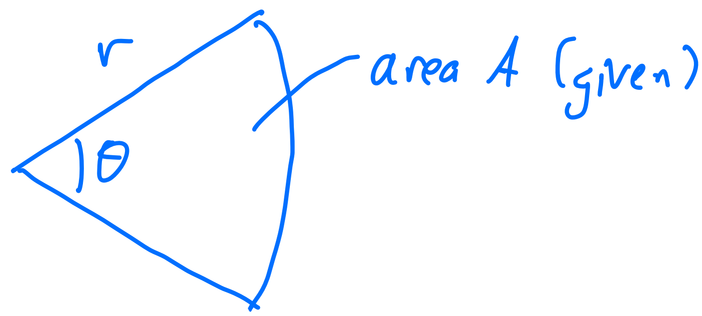
$$\underline{= \frac{1}{45}(x^3+4)^{15} + \cos x + K}$$

$$\frac{\int x^2\,dx}{x} = \frac{x^3/3 + c}{x} \qquad \frac{x^2}{3} + c + \cancel{x} \qquad c/x$$

3. (20 points)
A flower bed will be in the shape of a sector of a circle (a pie-shaped region) of radius r and vertex angle θ. Find r and θ if its area is a constant A and the perimeter is a minimum.

Area of circle: πr^2

angle $\theta = 2\pi$



Area of sector: $\pi r^2 \cdot \frac{\theta}{2\pi}$

$A = \frac{1}{2}\theta r^2$

Perimeter: Circumference of circle: $2\pi r$

length of arc of angle θ:

$2\pi r \cdot \frac{\theta}{2\pi} = r\theta$

$P = \underbrace{r\theta}_{\text{circ arc}} + \underbrace{r}_{\text{one radius}} + \underbrace{r}_{\text{another radius}} = r(\theta+2)$

minimize P such that area $= A$

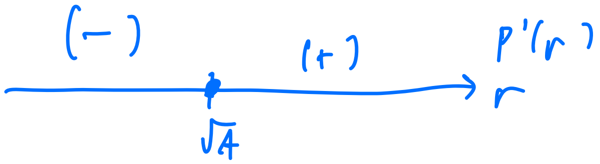
$A = \frac{\theta}{2} r^2 \Rightarrow \theta = \frac{2A}{r^2}$

$\Rightarrow P(\cancel{\theta}\, r) = r(\theta+2) = r\left(\frac{2A}{r^2} + 2\right) = 2r + \frac{2A}{r}$

$r > 0$

$P'(r) = 2 - \frac{2A}{r^2}$ Stationary pts: $P'(r) = 0 \rightarrow 2 = \frac{2A}{r^2}$

$r = \sqrt{A}$



$\Longrightarrow$ $r = \sqrt{A}$ is a local min.

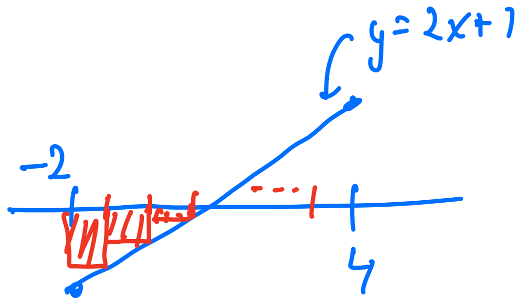
And since $p'(r) > 0$ for all $r > \sqrt{A}$, and $p'(r) < 0$ for all $0 < r < \sqrt{A}$, then $r = \sqrt{A}$ is actually a global min. location.

$$A = \frac{1}{2} r^2 \theta \xrightarrow{r=\sqrt{A}} A = \frac{1}{2} A \theta^{\cancel{2}} \Longrightarrow \theta = \cancel{\sqrt{2}}\ 2$$

$$(r, \theta) = (\sqrt{A}, \cancel{\sqrt{2}}\ 2)$$

4. (20 points)
Use the definition of the definite integral to compute

$$\int_{-2}^{4} (2x+1)\,\mathrm{d}x$$



Partition $[-2, 4]$ into n subintervals.

Width of single rectangle: $\frac{6}{n} = \Delta x$

x_0, $x_j = -2 + j\frac{6}{n}$, $j = 1 \dots n$

$x_0 = -2$ $\qquad x_n = -2 + 6 = 4$ ✓

On $[x_{j-1}, x_j]$, rectangle j, evaluate $f(\bar{x}_j)$ by taking $\bar{x}_j = x_j$

$$f(\bar{x}_j) = f(x_j) = f\left(-2 + j\frac{6}{n}\right) = 2\left[-2 + j\frac{6}{n}\right] + 1$$

$$= -3 + \frac{12}{n} j$$

n-rectangle Riemann sum: $R_n = \sum_{j=1}^{n} f(\bar{x}_j)\Delta x$

$$= \sum_{j=1}^{n} \left[-3 + \frac{12}{n} j\right] \frac{6}{n}$$

$$= \sum_{j=1}^{n} -\frac{18}{n} + \frac{72}{n^2} \underbrace{\sum_{j=1}^{n} j}_{\frac{n(n+1)}{2}}$$

$$= -\frac{18}{n}\cdot n + \frac{72}{n^2}\cdot\frac{n(n+1)}{2}$$

$$= -18 + 36\cdot\frac{n^2+n}{n^2}$$

$$= -18 + 36 + \frac{36}{n}$$

$$= 18 + \frac{36}{n}$$

$$\lim_{n\to\infty} R_n = \lim_{n\to\infty}\left(18 + \frac{36}{n}\right) = 18$$