Final Practice Exam
MATH1210, Section 023

Name: ______________________________

April 29, 2025

This test is:

- closed-book
- closed-notes
- no-calculator
- 120 minutes
- you have a separate reference sheet provided

Indicate your answers clearly, and show your work.

For question 1, multiple choice: answers are graded purely based on final answers.

For questions 2-6, free-response: Partial credit will be awarded based on work shown. Full credit will not be awarded without some work shown.

The multiple choice question is worth 50 points. Each free response question is worth 25 points. (175 points total)

Pages are two-sided. The first question begins on the back of this page!

1. (50 points total)

Multiple Choice. Record your final answers here: circle or mark with an X your alphabetic answers for each of parts (i) - (iv).

(i)	A	B	C	D	E
(ii)	A	B	C	D	E
(iii)	A	B	C	D	E
(iv)	A	B	C	D	E
(v)	A	B	C	D	E

(i) (10 pts) Suppose f is a twice differentiable function, and for some value c we have $f'(c) = 0$ and $f''(c) > 0$. Which of the following is always true?

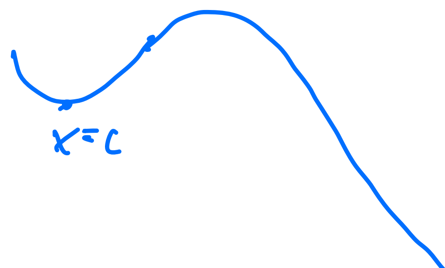
A. f has a local maximum at $x = c$

B. f has a local minimum at $x = c$

C. f has a global maximum at $x = c$

D. f has a global minimum at $x = c$

E. f has a point of inflection at $x = c$



(ii) (10 pts) Suppose F is an antiderivative of f. Which of the following is always true?

A. $f'(x) = F(x)$

B. $\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a)$

C. $F(x) - f(x) = c$ for some constant c

D. $f(x)$ is the signed area under the curve of F

E. F is undefined whenever $f(x) = 0$.

f is a derivative of F

$F'(x) = f(x)$

(iii) (10 pts) Suppose $f(x)$ is differentiable on $[a, b]$ and has a local maximum at some point c inside (a, b). Which of the following must be true?

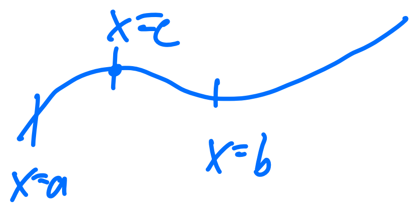
A. $\int_a^b f(x)\,\mathrm{d}x = 0$.

B. $f'(c)$ is undefined.

C. $f'(c) = 0$.

D. $\frac{f(b)-f(a)}{b-a} = f'(c)$

E. $\lim_{x\to c} f(x) = f(b) - f(a)$.



(iv) (10 pts) Suppose $x = c$ is a stationary point for a differentiable function $f(x)$. Which statement is always true?

$f'(c) = 0$

A. $f''(c)$ does not exist

B. $f(c)$ is the average value of f.

C. The slope of the tangent line to the graph at $x = c$ is undefined.

D. $f'(x) = f(c)$ for all x

E. $x = c$ is a candidate for the location of a local minimum or local maximum of f.

(v) (10 pts) Suppose $\lim_{x \to c} f(x)$ exists and equals L. Which of the following is always true?

A. f is differentiable at $x = c$.

B. The area under the curve of f at $x = c$ approaches L.

C. The value of $f(x)$ approaches L as x approaches c.

D. The average value of f is L.

E. f is continuous at $x = c$.

2. (25 points)
Consider the function $f(x) = x^4 - 2x^2 + 2$.

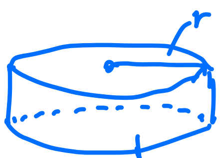
(i) (13 pts) Compute the global maximum and minimum values of f on the interval $[-2, 2]$.

(ii) (12 pts) Compute all points of inflection on $(-\infty, \infty)$ for f.

3. (25 points)
A circular metal disk expands during heating. If its radius increases at the rate of 0.01 centimeter per second, how fast is the area of one of its faces increasing when its radius is 10 centimeters?

$A = A(t)$

$r = r(t)$ $\quad r'(t) = 0.01$ [cm/s]



Area $A = \pi r^2$

What is $A'(t)$ when $r(t) = 10$ cm?

$$A'(t) = 2\pi\, r(t)\, r'(t)$$

$$= 2\pi \cdot 10 \cdot r'(t)$$

$$= 2\pi \cdot 10 \cdot 0.01$$

$$= \frac{2\pi}{10} = \pi/5 \text{ [cm}^2\text{/s]}$$

$$A = \pi r^2 \longrightarrow \frac{dA}{dr} = 2\pi r$$

$$\frac{dA}{dt} = \frac{d}{dt}(\pi r^2) = \pi \frac{d}{dt} r^2$$

$$= \pi \cdot 2r \cdot \frac{dr}{dt} = \pi\, 2 r r'$$

4. (25 points)

Consider the function $f(x) = \frac{x-4}{x+2}$

(i) (12 pts) Find the point(s) c guaranteed by the Mean Value Theorem for Derivatives applied to f on the interval $[-1, 1]$.

(ii) (13 pts) Find the point(s) c guaranteed by the Mean Value Theorem for Integrals applied to f on the interval $[0, 1]$.

$f(x) = \frac{x-4}{x+2}$

MVT for integrals: avg. value of f

$\shortparallel$

$\cancel{(b-a)} \cdot f(c)$ for some c

avg value of f: $\frac{1}{b-a} \int_a^b f(x)\,dx$

$a = 0, \quad b = 1$

Find c in $(0,1)$ s.t. $\frac{1}{1-0} \int_0^1 f(x)\,dx = f(c)$

$$\underbrace{\int_0^1 \frac{x-4}{x+2}\,dx}_{\text{In this class: can't compute this.}} = \frac{c-4}{c+2}$$

Suppose $K = \int_0^1 \frac{x-4}{x+2}\,dx$ (some number)

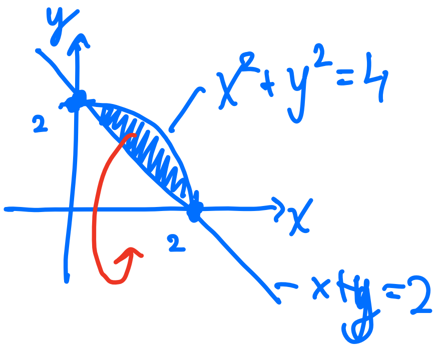
$$K = \frac{c-4}{c+2} \longrightarrow cK + 2K = c-4$$

$$c[K-1] = -4-2K$$

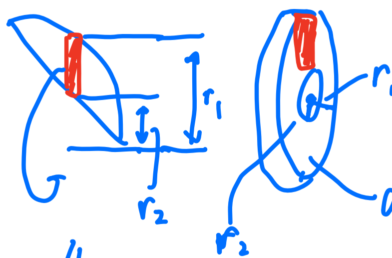
$$\boxed{c = \frac{4+2K}{1-K}}$$

5. (25 points)
Compute the area of the plane region bounded by the graphs of $x = y^2 - 2y$ and $x - y - 4 = 0$.

6. (25 points)
Find the volume of the solid generated by revolving about the x-axis the region in the first quadrant bounded by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$.



thickness Δx



$$\text{area} = \pi r_1^2 - \pi r_2^2$$

$$= \pi(4-x^2) - \pi(2-x)^2$$

$$= \pi(4-x^2) - \pi(4-4x+x^2)$$

$$= 4\pi x - 2\pi x^2$$

$\Downarrow$

$r_1 = y$-coord of circle
$= \sqrt{4-x^2}$

$r_2 = y$-coord of line
$= 2-x$

volume of washer: $(4\pi x - 2\pi x^2)\cdot \Delta x$

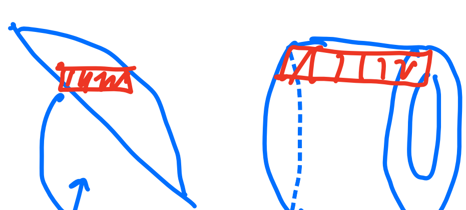
$$\text{Total volume: } \int_0^2 (4\pi x - 2\pi x^2)\,dx$$

$$= 2\pi \int_0^2 (2x - x^2)\,dx$$

$$= 2\pi \left[x^2 - \frac{x^3}{3}\right]\Big|_0^2$$

$$= 2\pi\left(4 - \frac{8}{3}\right) = 2\pi \cdot \frac{4}{3} = \frac{8\pi}{3}$$

Slice horizontally:



Volume of shell: (circumference) x (~~depth~~ thickness) x (width)

$$= 2\pi y \cdot \Delta y \left(\sqrt{4-y^2} - (2-y)\right)$$

Volume: $\int_0^2 2\pi y\left(\sqrt{4-y^2} - 2 + y\right) dy$