Practice Midterm Exam 1 **Name:** _____
MATH1210, Section 023 January 31, 2025

This test is:

- closed-book
- closed-notes
- no-calculator
- 50 minutes

Indicate your answers clearly, and show your work.

For question 1, multiple choice: answers are graded purely based on final answers.

For questions 2-4, free-response: Partial credit will be awarded based on work shown. Full credit will not be awarded without some work shown.

The multiple choice question is worth 40 points. Each free response question is worth 20 points. (100 points total)

Pages are two-sided. The first question begins on the back of this page!

1. (40 points total)
Multiple Choice. Record your final answers here: circle or mark with an X your alphabetic answers for each of parts (i) - (iv).

(i)	A	B	C	D	E
(ii)	A	B	C	D	E
(iii)	A	B	C	D	E
(iv)	A	B	C	D	E

(i) (10 pts) Suppose $\lim_{x\to 0^+} f(x) = 1$, and $\lim_{x\to 0^-} f(x) = 3$. Which of the following statements is true?

A. $\lim_{x\to 0} f(x) = 2$

B. $\lim_{x\to 0} f(x)$ exists, but cannot be computed with the given information.

C. $f(x)$ is discontinuous at $x = 0$.

D. $f(0) = 2$

E. All the above are false.

(ii) (10 pts) Identify any horizontal asymptote(s) that the function g below has.

$$g(x) = \frac{3x^2 - 9}{x^3 + 4x + 5}$$

A. $y = 3$

B. $y = 3$ and $y = 4$

C. $y = -9$

D. $y = 0$

E. No horizontal asymptotes

(iii) (10 pts) Suppose that $h(x)$ is continuous at $x = 1$ but is not differentiable at $x = 1$. Which of the following is NOT true?

A. $\lim_{x \to 1} \dfrac{h(x) - h(1)}{x - 1}$ exists

B. $\lim_{x \to 1} h(x)$ exists

C. $\lim_{x \to 1^-} h(x) = \lim_{x \to 1^+} h(x)$

D. $h(1)$ exists

E. All statements are true.

(iv) (10 pts) For a function $q(x)$, suppose that $q'(0) = 4$. Which of the following accurately interprets the statement $q'(0) = 4$?

A. $\lim_{x \to 0} q(x) = 4$

B. The instantaneous rate of change of q at $x = 4$ is 0.

C. The graph of q has a vertical asymptote at $x = 0$.

D. $q(0) = 4$

E. The slope of the tangent line to the graph of $q(x)$ at $x = 0$ is 4.

2. (20 points)
Compute the following limits. Answers may be numbers, DNE (does not exist), $+\infty$, or $-\infty$. (Stating DNE means the answer is <u>not</u> $\pm\infty$.)

(i) (5 pts) $\lim_{x\to 3} \dfrac{4}{3-x}$

(ii) (5 pts) $\lim_{x\to 2} \dfrac{x}{x^2+1}$

(iii) (5 pts) $\lim_{x \to 0} \dfrac{1}{|x|}$

(iv) (5 pts) $\lim_{x \to 0} \dfrac{1 - \cos x}{\sin(4x)}$

3. (20 points)
For this problem, define $f(x)$ as follows:

$$f(x) = \frac{|x-1|}{x^2-1}$$

See the guidance from question 2 for acceptable values of a limit.

(i) (10 pts) Determine the values of c for which $\lim_{x \to c} f(x)$ does <u>not</u> exist. For these values, compute the one-sided limits $\lim_{x \to c^+} f(x)$ and $\lim_{x \to c^-} f(x)$.

$$f(x) = \frac{|x-1|}{x^2-1}$$

(ii) (3 pts) Determine the value(s) of c for which f is discontinuous at $x = c$.

(iii) (7 pts) For the value(s) of c identified in (iii), identify which value(s) are removable discontinuities and which are non-removable discontuities.

4. (20 points)
For this problem, define $f(x)$ as follows:

$$f(x) = x^2 + 4$$

(i) (10 pts) Compute $f'(x)$ using the definition of the derivative.

(ii) (10 pts) Compute the equation of the tangent line to the graph of f at $x = 1$.