Practice Midterm Exam 2 **Name:** _____
MATH1210, Section 023 February 28, 2025

This test is:

- closed-book
- closed-notes
- no-calculator
- 50 minutes

Indicate your answers clearly, and show your work.

For question 1, multiple choice: answers are graded purely based on final answers.

For questions 2-4, free-response: Partial credit will be awarded based on work shown. Full credit will not be awarded without some work shown.

The multiple choice question is worth 40 points. Each free response question is worth 20 points. (100 points total)

Pages are two-sided. The first question begins on the back of this page!

1. (40 points total)
Multiple Choice. Record your final answers here: circle or mark with an X your alphabetic answers for each of parts (i) - (iv).

(i)	A	B	C	D	E
(ii)	A	B	C	D	E
(iii)	A	B	C	D	E
(iv)	A	B	C	D	E

(i) (10 pts) Suppose $f(1) = 2$, $g(1) = 0$, $f'(1) = 0$, and $g'(1) = 3$. Compute $h'(1)$, where $h(x) = f(x)g(x)$.

A. $h'(1) = 3$

B. $h'(1) = 2$

C. $h'(1) = 0$

D. $h'(1) = 5$

E. $h'(1) = -1$

(ii) (10 pts) Express the derivative $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{F(x)}\right)$ in terms of the function $F(x)$.

A. $\frac{1}{F'(x)}$

C. $\frac{-F'(x)}{F^2(x)}$

B. $\frac{-1}{F^2(x)}$

D. 0

E. $F'(x)$

(iii) (10 pts) Which of the following describes how one computes the maximum value of $f(x)$ on a closed interval I?

A. One finds the values x in I such that $f(x) = 0$, and computes the maximum of f over those values.

B. One finds the critical points of f on I and computes the maximum of f over those values.

C. One evaluates the maximum value of $f'(x)$ for x in I.

D. The maximum of f is the minimum of $f''(x)$, so one evaluates this latter quantity.

E. One finds the value x with maximum tangent line slope to the graph of f on I.

(iv) (10 pts) Suppose that a twice differentiable function $f(x)$ satisfies $f'(x) < 0$ and $f''(x) > 0$ for x in $I = (0, 3)$. Which of the following must be true?

A. f is increasing on I and concave up on I.

B. f is increasing on I and concave down on I.

C. f is decreasing on I and concave up on I.

D. f is decreasing on I and concave down on I.

E. $f(x) < 0$ for x in I.

2. (20 points)

(i) (4 pts) Compute $y'(x)$ if $y(x) = \sin(x^3)$

(ii) (4 pts) Compute $y'(x)$ if $y(x) = \frac{x^2}{\cos x}$

(iii) (8 pts) If $y \sin x = \cos(xy)$, compute $y'(x)$ as a function of x and y.

(iv) (4 pts) Compute the equation of the tangent line to the graph of $y(x)$ in the previous part (iii) at the point $(x, y) = \left(\pi, \frac{1}{2}\right)$.

3. (20 points)
A woman standing on a cliff is watching a motorboat through a telescope as the boat approaches the shoreline directly below her. If the telescope is 250 feet above the water level and if the boat is approaching at 20 feet per second, at what rate is the angle of the telescope changing when the boat is 250 feet from the shore?

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4. (20 points)
For this problem, define

$$f(x) = \frac{1}{1+x^4}$$

(i) (5 pts) Determine the critical points of f.

(ii) (5 pts) Determine where f is increasing and decreasing, with justification.

(iii) (5 pts) Determine where f is concave up or concave down, with justification.