

In all that follows,  $a, b, c, c_1$  and  $c_2$  are arbitrary real constants, and  $f$  and  $g$  are functions.

$n$  is any positive integer.

$r$  is any rational number.

The notation  $\pm$  means “plus or minus”, and  $\mp$  is “minus or plus”. When used in the same equations, these symbols retain their ordering. E.g.,  $\pm x = \mp 3$  means both “ $+x = -3$ ” and “ $-x = +3$ ”.

### Trigonometric identities and alues:

$$\begin{aligned}\sin(a \pm b) &= \sin a \cos b \pm \cos a \sin b & \cos(a \pm b) &= \cos a \cos b \mp \sin a \sin b \\ \sin(2a) &= 2 \sin a \cos a & \cos(2a) &= \cos^2 a - \sin^2 a \\ \sin^2 a + \cos^2 a &= 1 \\ \sin 0 &= \cos \frac{\pi}{2} = 0 & \sin \frac{\pi}{2} &= \cos 0 = 1 \\ \sin \frac{\pi}{6} &= \cos \frac{\pi}{3} = \frac{1}{2} & \sin \frac{\pi}{3} &= \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \\ \sin \frac{\pi}{4} &= \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}\end{aligned}$$

### Limits and continuity:

$$\begin{aligned}\lim_{x \rightarrow c} c_1 &= c_1 & \lim_{x \rightarrow c} x &= c \\ \lim_{x \rightarrow c} (c_1 f(x) \pm c_2 g(x)) &= c_1 \lim_{x \rightarrow c} f(x) \pm c_2 \lim_{x \rightarrow c} g(x) & \lim_{x \rightarrow c} f(x)g(x) &= \left( \lim_{x \rightarrow c} f(x) \right) \left( \lim_{x \rightarrow c} g(x) \right) \\ \lim_{x \rightarrow c} \frac{f(x)}{g(x)} &= \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} g(x)} \quad (\text{Assuming the denominator is not } 0) \\ \lim_{x \rightarrow 0} \frac{\sin x}{x} &= 1 & \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} &= 0 \\ \lim_{x \rightarrow c} f(g(x)) &= f(g(c)) \quad (\text{Assuming } f \text{ is continuous at } \lim_{x \rightarrow c} g(x).)\end{aligned}$$

A function  $f$  has a vertical asymptote at  $x = c$  if  $\lim_{x \rightarrow c} |f(x)| = \infty$ .

A function  $g$  has a horizontal asymptote at  $y = c$  if  $\lim_{x \rightarrow +\infty} g(x) = c$  or  $\lim_{x \rightarrow -\infty} g(x) = c$ .

### Derivatives:

If  $f$  is differentiable at  $x = c$ , then  $f$  is continuous at  $x = c$ .

$$\begin{aligned}(x^r)' &= rx^{r-1}, & (c_1 f(x) + c_2 g(x))' &= c_1 f'(x) + c_2 g'(x) \\ (f(x)g(x))' &= f'(x)g(x) + g'(x)f(x) & \left( \frac{f(x)}{g(x)} \right)' &= \frac{f'(x)g(x) - g'(x)f(x)}{g^2(x)} \\ (\sin x)' &= \cos x & (\cos x)' &= -\sin x \\ (\tan x)' &= \sec^2 x & (\cot x)' &= -\csc^2 x \\ (f(g(x)))' &= f'(g(x))g'(x)\end{aligned}$$

Critical points of functions over a closed interval are comprised of the interval endpoints, stationary points, and singular points.

Newton’s Method for numerically solving  $f(x) = 0$  corresponds to the iteration  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ .

**Sums and integrals:**

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^n i^3 = \frac{(n(n+1))^2}{4}$$

If  $F$  is an antiderivative of  $f$ , then the substitution rule for integrals is,

$$\int f(g(x))g'(x) \, dx = F(g(x)) + C$$

**Areas and volumes:**

$$\text{Area of } \theta\text{-sector of a circle} : \frac{\theta}{2} \times r^2 = \frac{\theta}{2} \times (\text{radius})^2$$

$$\text{Circular arc length sweeping out } \theta \text{ radians} : \pi \times \theta$$

$$\text{Volume of a sphere} : \frac{4}{3}\pi r^3 = \frac{4\pi}{3} (\text{radius})^3$$

$$\text{Volume of a thin cylinder} : A \times \Delta h = (\text{Area of base}) \times (\text{height})$$

$$\text{Volume of a thin shell} : 2\pi r \times h \times \Delta r = (\text{Shell circumference}) \times (\text{height}) \times (\text{thickness})$$

**Graphs:**

- Point-slope form for a line:  $y - y_0 = m(x - x_0)$
- Equation of a circle:  $(x - x_0)^2 + (y - y_0)^2 = r^2$