Math 1210: Calculus I
Limits for trigonometric functions

Department of Mathematics, University of Utah

Spring 2025

Accompanying text: Varberg, Purcell, and Rigdon 2007, Section 1.4

Trigonometric functions

With a firmer understanding of trigonometric functions, we can delve into some limits involving these functions.

Example

Show that $\lim_{t \to 0} \sin t = 0$, and $\lim_{t \to 0} \cos t = 1$.
(Squeeze between 0 and t.)

Example

Show that $\lim_{t \to c} \sin t = \sin c$.
(Substitute $t = c + h$, use angle sum formula)

A more general result

We have seen that limits of the sine function can be evaluated through substitution. This is a generic property of trigonometric functions.

Theorem (Substitution for trigonometric limits)

As long as c is in the domain of the functions below, then,

$$\lim_{t \to c} \sin t = \sin c \qquad \lim_{t \to c} \cos t = \cos c \qquad \lim_{t \to c} \tan t = \tan c$$

$$\lim_{t \to c} \csc t = \csc c \qquad \lim_{t \to c} \sec t = \sec c \qquad \lim_{t \to c} \cot t = \cot c$$

Some special limits

With rational functions, if we encountered $\frac{0}{0}$ when performing substitution for a limit, we attempted to rectify this by factoring and canceling terms.

We cannot do something quite so simple with trig functions. However, there are some special limits to note that will be very useful.

Some special limits

With rational functions, if we encountered $\frac{0}{0}$ when performing substitution for a limit, we attempted to rectify this by factoring and canceling terms.

We cannot do something quite so simple with trig functions. However, there are some special limits to note that will be very useful.

Theorem (Special trigonometric limits)

The following limits hold:

$$\lim_{t \to 0} \frac{\sin t}{t} = 1, \qquad \lim_{t \to 0} \frac{1 - \cos t}{t} = 0.$$

Some special limits

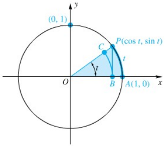
With rational functions, if we encountered $\frac{0}{0}$ when performing substitution for a limit, we attempted to rectify this by factoring and canceling terms.

We cannot do something quite so simple with trig functions. However, there are some special limits to note that will be very useful.

Theorem (Special trigonometric limits)

The following limits hold:

$$\lim_{t \to 0} \frac{\sin t}{t} = 1, \qquad \lim_{t \to 0} \frac{1 - \cos t}{t} = 0.$$



Let's show the first limit:

Figure 2

Evaluating limits

For limits involving polynomials, rational functions, root functions, or trigonometric functions, here is an ordered set of steps for evaluation:

1. Direct substitution: if the result is a valid number, that is the limit.
2. Rational cancellation: Attempt to factor + cancel terms to remove behavior like $\frac{0}{0}$
3. Use special trigonometric limits for $\frac{0}{0}$-like behavior stemming from trig functions

Examples

Example

Evaluate $\lim_{x \to 0} \frac{\sin 2x}{x}$.

(Ans: 2)

Example

Evaluate $\lim_{x \to 0} \frac{1 - \cos t}{\sin t}$.

(Ans: 0)

Examples

Example

Evaluate $\lim_{x \to 0} \frac{\tan x}{\sin 3x}$.

(Ans: $\frac{1}{3}$)

References I

Varberg, D.E., E.J. Purcell, and S.E. Rigdon (2007). *Calculus*. 9th. MyMathLab Series. Pearson Prentice Hall. ISBN: 978-0-13-142924-6.