A **AN EXAMPLE OF MERGE TREE**

We now give an example of an augmented merge tree that is constructed from the ERA5 Wind dataset. First, we define a scalar field \( f_0 : \mathbb{X} \rightarrow \mathbb{R} \) by assigning the vector magnitude to each point \( x \in \mathbb{X} \), that is, \( f_0(x) = ||f(x)||_2 \); see Fig. 12(A) and (B), which visualize the vector field \( f \) and scalar field \( f_0 \). Second, we track the merging behavior of components that contain critical points of \( f \). For example, the component containing \( x_1 \) and \( x_2 \) merges with the component containing \( x_3 \) at \( r = 0.84 \) and forms \( C_3 \) in \( \mathbb{X}_{0.84} \), which is represented by the purple and green region in Fig. 12(D). Third, we augment the merge tree with the degrees of critical points (on leaves) and the degrees of components (on internal nodes). For example, component \( C_1 \) in Fig. 12(D) contains critical points \( x_1 \) and \( x_2 \), whose degrees are \( +1 \) and \( -1 \), respectively. The degree of \( C_1 \) is \( \text{deg}(x_1) + \text{deg}(x_2) = 0 \), i.e., \( \text{deg}(C_1) = 0 \). The augmented merge tree of Fig. 12(A) is shown in Fig. 12(E).

B **AN EXAMPLE OF ROBUSTNESS CALCULATION**

Using the example in Fig. 12(A)-(E), we now show how to calculate robustness with an augmented merge tree. As pointed out in Sec. 3.1, the robustness of a critical point can be calculated as the function value of its lowest zero-degree ancestor in the augmented merge tree. The robustness of \( x_1 \) and \( x_2 \) is 0.65, whereas the robustness of \( x_3 \) and \( x_4 \) is 14.7. Intuitively, for the example in Fig. 12(A)-(E), it is easier for \( x_1 \) and \( x_2 \) to be canceled with each other than \( x_3 \) and \( x_4 \), since they have much lower robustness values. In Fig. 12(F), we give the vector field from the same dataset but one time step (6 hours) behind the vector field of Fig. 12(A). We see \( x_1 \) and \( x_2 \) disappear, whereas \( x_3 \) and \( x_4 \) remain.

![Fig. 12: Computing robustness with an augmented merge tree. (A) A 2D vector field \( f \) and (B) its corresponding scalar field \( f_0 \). (C) Multiple sublevel sets of \( f_0 \). (D) A zoomed-in view of the white box in (C). (E) The augmented merge tree. (F) The vector field one time step behind (A). Sources/sinks/centers are in red, and saddles are in blue.](image)

C **DETAILS ON DATASET AND METHODS**

We demonstrate the performance of TROPHY using 30-year (1981–2010) near-surface wind vector field from the ECMWF Reanalysis v5 (ERA5). It is produced by the Copernicus Climate Change Service (C3S) [1]. ERA5 provides hourly estimates of the global climate information with a spatial grid resolution of 30 km. Since tropical cyclones/storms usually occur during June and October, we limit our dataset with a time window from June 1 to October 31 every year at standard synoptic reporting times (0000, 0600, 1200, and 1800 UTC). A rectangle region on the Atlantic Ocean (5 N° to 49.5 N° and 98 W° to 18 W°) is selected. We utilize 10-meter zonal and meridional wind speed as the 2D vector field, since in the near-surface the hurricane core represents a region of strong convergence and associated vertical motion. We annotate this 30-year dataset as the ERA5 Wind dataset. We also mark the one-year subset data from the ERA5 Wind dataset as ERA5 Year; for example, Fig. 3 uses the ERA5 2004 dataset.

We use the International Best Track Archive for Climate Stewardship (IBTrACS [26] version 4) observations as the reference. IBTrACS is compiled from quality-controlled records from various forecasting