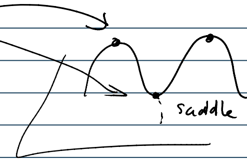
Class 6

Monday, 16 February 2015 6:05 AM

Function $D \to R$

Critical points $Df = 0$

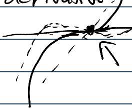


a function is Morse if

+ all CP have distinct function values
(x, y are critical $\to f(x) \neq f(y)$)

not having plateu

+ second derivative is non 0 (Hessian Matrix is full rank)

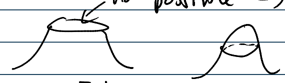


← eliminate this case

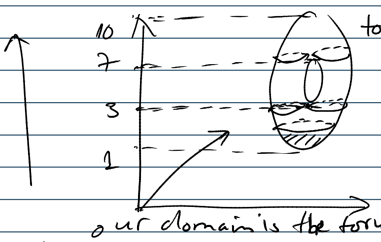
not allowed (we can always have a Morse function arbitrarily close to an invalid function)

Level set: $L(\omega)$ = set of points x where $f(x) = \omega$
(or $f^{-1}(\omega)$)

if $D \subseteq R^2$ then $DIM(f^{-1}(D)) \leq 1$
not possible ↳ and most of the time it is 1



Sub-levelset $L^-(\omega)$: $f(x) \leq \omega$



this sweeping is like a filtration (see last lecture)

our domain is the torus

Betty numbers of the sub-level sets:

$L^-(\omega)$ when $\omega < 1$:
$B_0 = B_1 = B_2 = 0$

$L^-(\omega)$ when $\omega = 1$
$B_0 = 1$ $B_1 = 0$ $B_2 = 0$

$L^-(\omega)$ $1 < \omega < 3$
$B_0 = 1$, $B_1 = 0$, $B_2 = 0$

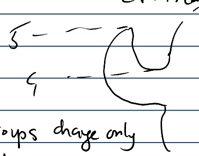
$L^-(\omega)$ $\omega = 3$ (create new cycle)
$B_0 = 1$, $B_1 = 1$, $B_2 = 0$

$L^-(\omega)$ $3 < \omega < 7$
$B_0 = 1$, $B_1 = 1$, $B_2 = 0$

$L^-(\omega)$ $\omega = 7$ (creat new cycle)
$B_0 = 1$, $B_1 = 2$, $B_2 = 0$

$L^-(\omega)$ $\omega = 10$ (creat new 2-cycle (like adding a closing triangle)
$B_0 = 1$, $B_1 = 2$, $B_2 = 1$

* if now we introduce another critical point at 5,



at 4: $B_1 + 1$
at 5: $B_1 - -$
this creates a homology group and then destroys it

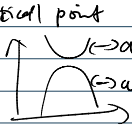
the homology groups change only at critical points

we can analyze homology groups that emerge in time and find large-scale and small-scale structures in time

+ adding a critical point is like adding an vertex / edge / triangle in the filtration case
(e.g. adding maximum == adding a triangle)

* index of critical point $f(x) = a_0 x_0^2 + a_1 x_1^2 + \dots +$ (local expansion of f)

(think of Taylor series, by first translate f so that $f(x) = 0$) ↳ $f' = 0$ so first term gone, then transform the function to make $a_i = \pm 1$



$\leftrightarrow a_i = 1$

$\leftrightarrow a_i = -1$

$a_i = +1$ or -1

index $= m - K$ → count the number of a_i that is $+1$

so for the minimum point, index $= 0$

maximum point, index $= m$

also, index = # of negative eigen values

direction of eigen vectors

saddle point = some eigen vectors go ↑ some ↓

adding a minimum = same as adding 0-simplex
a 1-sadde = ______ 1-simplex (edge)
a 2-sadde = ______ 2-simplex (triangle)
a 3-sadde = ______ 3-simplex (tetra)

depends on the function