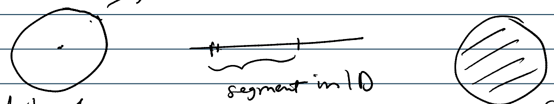
Class 2

Thursday, 29 January 2015 4:04 AM

Manifold → simplest 1D manifold

. ← a point is a 0-manifold



disk of — sphere in 3D including the interior

LOCAL property: radius ≤ 1

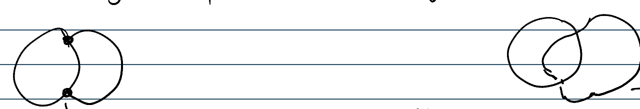
(n-ball in any dimension)

$|x_i| = 1$ → n-sphere

manifold = object where all points look like an n-ball

for every point, there is a neighborhood that looks like a segment (in 1D)

example:



→ T shape → not manifold

→ intrinsic dimension 1 (1-manifold)

↓

can be embedded using a bijective function (attach a unique continuous coordinate)

intrinsic vs embedding (example: a surface is a 2D object but can be put into 3D)

↓

2D, 3D space

⊛ half disk : → manifold with boundary is NOT manifold $|x| < \varepsilon$ & $x_0 \geq 0$ (boundary case)

⊛ 1-manifolds can not be embedded in 1D. (true for n-dimensions) — they are just a bunch of circles

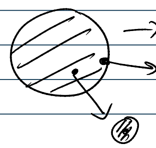
↓

R^n is an open space while manifold is "closed"

⊛ sphere vs disk / ball

$|x| = 1$ $|x| \leq 1$

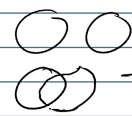
2-sphere is the boundary of 3-disk



→ 2-manifold with boundary

→ ⊛ manifold is a special case of manifold with boundary

⊛ klein bottle is a 2-manifold but can only be embedded in 4D because it does not enclose a space in 3D



→ can be embedded in 2D

→ same manifold but can only be embedded in 3D

⊛ orientability :

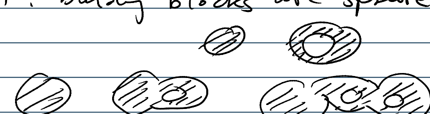
walking along the object to see if the normals are flipped

the definition based on normals is bad because it depends on embedding

a mobius strip is not orientable → global property but can be derived locally

nor any

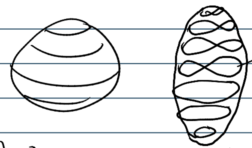
orientable 2-manifold : building blocks are sphere and torus



in 2D : $\underset{c}{O}$ $\underbrace{O\ O}_{cc}$ $\underbrace{OOO}_{ccc}$

non-orientable 2-manifold : projective plane c cc ccc

non-orientable 2-manifold : projective plane



cross cap

(similar to a dish but is a mobius strip)

simplest non orientable 2-manifold: cross cap

— Question : is it possible to make a non-orientable 1-manifold?