Class 5

Wednesday, 11 February 2015 6:04 AM

Object embedded in 3-sphere (S^3)

↳ kind of a proxy of R^3

Filtration : a sequence of simplices (can be of different dimension) such that any prefix of this sequence is a complex

* this is like having a set of points and start adding edges to connect these points
* review of complex :

simplex: $\{S_0, \ldots S_n\}$

Complex: + $S_i \cap S_j = S_k \Rightarrow \begin{cases} S_k \subseteq S_i \\ S_k \subseteq S_j \end{cases}$

an abstract simplex is a set of symbols

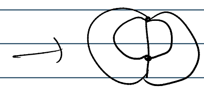
+ if $S_k \subseteq S_i$, $S_i \in C$ $\Rightarrow S_k \in C$

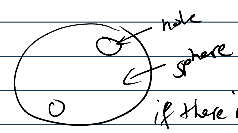
We are interested in B_0, B_1, B_2 for a complex in 3D

the set of simplices we are interested in are $\begin{cases} \text{vertex} \\ \text{edge} \\ \text{triangle} \\ \text{tetrahedra} \end{cases}$

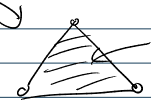
- algorithm: sort the simplexes and add them one by one

so when:

+ we add a vertex: B_0++
+ we add an edge: if it connects 2 components B_0-- ; if not, B_1++ →  ?
+ we add a triangle: either B_2++ or B_1--



if there is 2 holes there is 1 cycle



when the triangle is added the cycle is gone (B_1--)

* split the next dimension in 2 pieces

+ we add a tetrahedra: B_3++ / B_2--

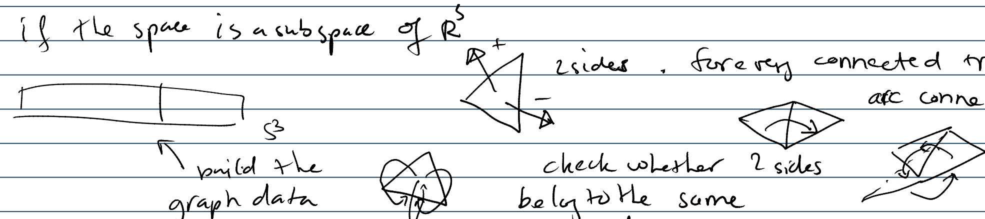
(*) all triangle / tetrahedra (except the last) will decrease the last Betty number, the last one will close the space and increase the next Betty number (important: we assume our filtration in the end is S^3)

0 easy
1 union find
2 going backwards on the filtration, running union find
3 easy

↓ if a triangle triggers the union operation, B_2++
↳ to check this we need to query the sets to which the 2 tetrahydra on its sides connect (find operation)

this is not an online algorithm anymore

(*) if the space is a subspace of R^5



2 sides. for every connected triangles are connect

build the graph data structure at this point from the mesh

check whether 2 sides belong to the same connected components ✓

3 components

we are still doing the algorithm in reverse point from the ...

can only be done in 3D
not symbollically

Proof: Boundary of boundary is 0 / S $\{v_0 \ldots v_n\}$

$n+1$ simplices $\{v_0, \ldots \hat{v}_k, \ldots, v_n\}$

for each of those, we have n $\{v_0 \ldots \hat{v}_{k}, \ldots, \hat{v}_j, \ldots v_n\}$

2 ways to remove $\hat{v}_k$ $\hat{v}_j$

$$\sum_{k,j} = \sum 2S_1 + 2S_2 \ldots = 0$$

every simplex two dimensions down just creates 2 copies of the same thing