Class 3

Monday, 2 February 2015 3:54 AM

topology talks about open set

$S_1, \dots, S_n$ are open subsets of X

$S_i \subset X$

1) $X, \emptyset$ are open

2) if S_1, S_2 are open $\rightarrow$ $S_1 \cap S_2$ are open $\subseteq S$ (intersection) must be finite

3) $\bigcup_i S_i$ is open (union)

connectedness: if $X = S_1 \cup S_2$ and $S_1 \cap S_2 = \emptyset$ then X is not connected

Example: $\mathbb{R}$ ——( )——→

(a, b) set of all open segments is a topological space

$C_i = X \setminus S_i$ is a closed set

$(-\infty, a] \cup [b, +\infty)$ is a closed set

$(-\infty, a) \cup (b, +\infty)$ is an open set

$(a, b]$ is neither open nor closed

$X, \emptyset$ is both open and closed

a set can be – open / closed / both / neither

intersections of infinitely many open set can be a closed set (e.g. $(-\frac{1}{i}, \frac{1}{i})$)

→ so we need a finite number of sets when doing intersection.

* Homeomorphism (NOT homomorphism)

1-1 Mapping from X to Y that is continuous & the inverse is continuous

Example (—o—) $-1 \quad 0 \quad 1$

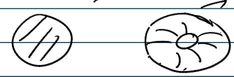
is 1-disk and $\mathbb{R}$ the same thing? $\left(f(x) = \frac{1}{x-1} + \frac{1}{1+x}\right)$

– an object is a 1-manifold is at any point on the object there is a neighborhood that is homomorphic to a 1-disk.

– a soccerball without the interior is a 2-manifold (because of homomorphic to an open disk in a neighborhood)

→ not topologically the same

* 2 manifold:



sphere torus

(think about a circle hugging the surface that can shrink to a point)

can link toruses to create other 2-manifold

* Simplex $\{v_0, v_1, \dots v_n\}$ form the vertices of a simplex if they are independent / convex combination

$\{v_0\}$ = a point

$\{v_0, v_1\}$ = 1-simplex (edge)

(triangle)

(tetrahydra)

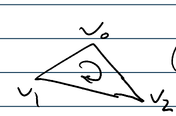
a complex $C = \{S_0, S_1, \dots S_n\}$

if a simplex is part of a complex, each of its faces is also part of the complex

↓

obtained by removing vertices

Order:



$(v_0, v_2, v_1) \neq (v_0, v_1, v_2)$

↑

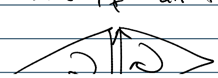
flipping the orientation of two adjacent vertices

if we remove a vertex: $(v_0, v_2) = \xrightarrow[v_0 \quad v_2]{}$

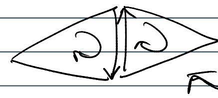
$(v_2, v_0) = \xleftarrow[v_0 \quad v_2]{}$

$-1^i (v_0, \dots v_{i-1}, v_{i+1}, \dots, v_n)$

a manifold is orientable if all the simplexes can be oriented so that common faces between adjacent simplexes are opposite

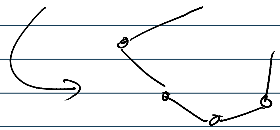


a manifold is orientable if all the simplexes can be oriented so that common faces between adjacent simplexes are opposite



example: the common edge has two opposite orientation

Q can we have a non-orientable 1-manifold?



(v_1, v_2) (v_2, v_3) ...

faces $= -(v_1), (v_2)$, $-(v_2)(v_3)$

since the face is only a point, there is only one orientation (as compared to an edge for example)

cannot have more orientations