CS6210 Homework Set 1 (Fall 2010)

1. (20 pts) Implement in single precision and test the HS (high school) and MA (modified) algorithms for computing roots of quadratic equations $x^2 - 2px + q = 0$ with p and q nonzero and satisfying $p^2 - q > 0$. Find two cases of coefficients p and q where in each case one of the roots computed via the HS algorithm is of poor quality. You may compute the "exact" roots by using MA in double precision and use these roots to compare the actual relative errors with expected relative errors obtained through formulas developed in lecture. Please display all your results in easy to read tabular form.

2. (20 pts) Show that Horner's Algorithm is Numerically Well Behaved (N.W.B.). Apply your analysis to an arbitrary polynomial $p(x)$ of degree n of the form $p(x) = \sum_{i=0}^{n} a_i x^i$.

3. (20 pts) The polynomial $p(x) = (x - 1.0)^6$ has the value 0 at $x = 1$, and is positive elsewhere. The expanded form of $p(x)$ is $p(x) = x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1$.

Compute and plot the values of $p(x)$ using each of the three forms:
(a) One subtraction and 3 multiplications,
(b) Horner's algorithm,
(c) trivial summation of the terms in $p(x) = x^6-6x^5+15x^4-20x^3+15x^2-6x+1$;
In all cases take 101 uniformly spaced points in the interval $[0.995, 1.005]$ (hence with spacing 10^{-4}). Now explain the behavior of the computed values of $p(x)$.

4. (10 pts) Propose an algorithm to compute the expression $w(x, y) = \frac{1}{(x-8)y} + \frac{1}{(x+12)y}$ such that the relative error of the computed result δ satisfies $|\delta| \leq C2^{-t}$ and is such that C is as small as possible. You may assume that x and y are machine numbers.

5. (30 pts) Compute e^{-x} for $x = 2, 4, 6, 8, 10, 12, 15, ...$ by using:

(a) $e^{-x} = \sum_{i=0}^{\infty} \frac{(-x)^i}{i!}$; (b) $e^{-x} = \frac{1}{e^x}$ and $e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!}$; (c) Standard function $\exp(-x)$.

First compare and then explain your results.

Helpful Comments:

(i) When computing infinite summations $\sum_{i=0}^{\infty} a_i$, you should terminate your algorithm whenever
$fl(S_n + a_{n+1}) = S_n$, where $S_n = fl(\sum_{i=0}^{n} a_i)$ for $a_i \to 0$.

(ii) When providing explanations strive to frame them using terms and concepts presented and developed in lecture.

(iii) Please include and collect at the end of your homework all your relevant computer code.