CS6210 Homework Set 2 (Fall 2010)

1. (20pt) Carry out floating point analysis of the BSA algorithm for the solution of an upper triangular system. Conclude that it is numerically "Well Behaved" algorithm.

2. (80pts)

For $n = 3, 4, \ldots, 10$ let L_n be the lower triangular matrix with all 1's and let U_n be the upper triangular matrix with all 2's on the diagonal and all 1's for off diagonal elements. Since the determinant of a triangular matrix is the product of its diagonal elements, and the product of invertible matrices is invertible, for each n matrix $P_n = L_n U_n$ is invertible. We define matrices $A_n = P_n P_n^T$, the product of P_n with its transpose.

(0) Why is A_n SPD? (Hint: Use the fact that for any $n \times n$ matrix M and any vectors $x, y \in \mathcal{R}^n$,

$$< x, My > = < M^T x, y >$$

where inner-product $< x, y >$ equals $x^T y$ and gives $||x||_2^2 = < x, x >$.)

In single precision test your Cholesky and Householder algorithms for $A_n x = b$, $n = 3, 4, \ldots, 10$ by setting $x = [1, 1, \ldots, 1]^T$ and computing the appropriate corresponding vector b. First apply the Cholesky and Householder algorithms and then follow it by three steps of iterative refinement.

(1) Compare your computed solution $\widetilde{x}$ with the exact solution $x = [1, 1, \ldots, 1]^T$. Explain your results.

(2) It is essential that you employ all the computational shortcuts discussed in lecture with respect to storing and multiplying Householder matrices.

(3) Compute residual vectors in Iterative Refinement using double precision.

(4) Display all your numerical outputs in easy to read and easy to decipher tables.

Repeat the above when A is the $n \times n$ Hilbert matrix defined as $A = [a_{ij}]_{i,j=1}^n$ with element a_{ij} equal to $(i + j - 1)^{-1}$.