CS6210 Homework Set 3 (Fall 2010)

1. (100 pt) Implement in double precision the Gauss-Seidel, SOR, Chebyshev and CG methods for coding the two dimensional Poisson's Problem with known solution to $u_{xx}(x,y) + u_{yy}(x,y) = f(x,y)$ on the unit square $[0,1]^2$ equal to

$$u(x,y) = 128 \cdot x(1-x)y(1-y).$$

You can see then that we are also assuming boundary conditions $u = 0$ along the perimeter of the square.

First, determine function $f(x,y)$ that corresponds to the above solution. Next, test your code for $n = 10, 100, 1000$ (or possibly higher if you dare!). Then compare computed solution $\widetilde{u}$ to the exact solution u. You should terminate when u and $\widetilde{u}$ satisfy $||\widetilde{u} - u||_\infty \leq \epsilon$ for small $\epsilon > 0$ appropriately chosen, as discussed in class.

We recall that optimal ω for SOR equals $\dfrac{2}{1 + \sin \frac{\pi}{n+1}}$.

Successful submissions will

1. Provide careful derivation of forcing function f.
2. Make known the epsilon value(s) used and why chosen.
3. Display in a single table for each of the above algorithms values of:

$$\max_{i,j=1,\ldots,n} |\widetilde{u}(x_i, y_j) - u(x_i, y_j)|$$

where $x_i = i \cdot h$, $y_i = j \cdot h$, $h = 1/(n+1)$.

4. Fill in as many positions as possible in the following table including in each case the associated value of ϵ. CPU times (CPU) should be in seconds. Numbers in parentheses refer to values for n.

Method	#Iter (10)	CPU (10)	#Iter (100)	CPU (100)	#Iter (1000)	CPU (1000)
G-S						
SOR						
Chebyshev						
CG						

5. Based on formulas presented in class involving eigenvalues and condition numbers, compare and justify expected and actual differences in CPU times and #-iterations among the various algorithms.