CS5960: Nonparametric Methods **Due Wed. 3/4**

Homework 4: Monte Carlo and Variance Reduction

Instructions: Be sure to electronically submit your answers in pdf format for the written part and as an R file for the coding part. You may work together and discuss the problems with your classmates, but write up your final answers entirely on your own.

1 Written Part

For the problems below consider the integral

$$\theta = \int_0^1 e^{-x} dx.$$

1. What is the expected value and variance of the simple Monte Carlo estimate, $\hat{\theta}$?
2. Now consider the stratified Monte Carlo estimate $\hat{\theta}^S$ that has two strata, $[0, 0.5]$ and $[0.5, 1]$. What is the variance of $\hat{\theta}^S$? What is the percent reduction in variance over the simple Monte Carlo estimate $\hat{\theta}$?
3. What is $\text{Var}\left(e^{-U} + e^{-(1-U)}\right)$, where $U \sim \text{Unif}(0, 1)$? What would be the variance of a Monte Carlo estimate of θ using antithetic variables? What is the percent reduction in variance over the simple Monte Carlo estimate $\hat{\theta}$?

2 R Simulation

4. Write R code to generate the Monte Carlo estimates of θ using the three methods in Problems 1-3. Generate 1000 different estimates, each using 1000 random numbers, and compute the variance of your estimators.
5. Write R code that uses importance sampling to estimate the integral

$$\theta = \int_0^\infty \frac{x^2}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx.$$

   Try two different importance functions and compare empirically which has the lowest variance (again using 1000 estimates, 1000 random numbers).