CS 7960, Advanced Image Processing
Spring 2010, Prof. Guido Gerig

Project 4: Active Shape Models ASM

Out:	Thursday Mar-11-2010
Due:	Thursday April-01-2010
Office hours:	Tue 1pm to 3pm, please contact me in advance or ask for other arrangements.

Required Readings: Papers and Materials Lectures on ASM (Papers and Slides by Cootes and Taylor.
Mikkel Stegman and David Delgado Gomez, DIKU, provide an excellent short description of ASM, available at $http://www2.imm.dtu.dk/pubdb/views/edoc_download.php/403/pdf/imm403.pdf$).
We did not yet discuss the shape alignment and registration as covered in the first half of this document, and this is not necessary for this project.

ASM

Let us consider a set of shapes given by shape vectors $\vec{x} = (x_1, y_1, x_2, y_2, \cdots, x_n, y_n)^T$, which represents a Point Distribution Model (PDM).
These vectors are input to a principal component analysis (PCA) to determine the principal shape variations of the population of shapes. The eigenvalues determine the magnitude of the variation whereas the eigenvectors describe the new subspace. By truncation to the major set of eigenvalues with corresponding eigenvectors, shapes are represented by a small set of coefficients which represent smoothed, simplified versions of shapes and second are often used for classicfication and object detection.

Instructions

You need to implement a statistical shape model based on the point distribution model (PDM) developed by Cootes and Taylor.
Define a simple shape, e.g. rectangular shape with 4 corners or any more interesting shape of your choice. Generate a family of shapes by either varying the size, and/or adding translations, stretching or other transformations in a random way. Apply a PCA to these shape families and discuss the eigenvalues. You can also display the major modes of shape variation by synthesizing shapes deviating from the mean along each new shape eigenmode axis i, defined by a factor s applied to the standard deviations which can range from -2 to $+2$, for example:

$$PCshape[i_-, s_-] := Mean + s * Sqrt[eigval[[i]]] * eigvec[[i]].$$

Also, we would like to visualize the correlation of pairs of point coordinates by converting the covariance matrix into a *correlation matrix* and displaying this square matrix as an image (please note that this matrix shows positive and negative values, so proper scaling to a positive range is required). The correlation matrix is a square symmetrical MxM matrix with the (ij)th element equal to the correlation coefficient $r_i j$ between the (i)th and the (j)th variable. The diagonal elements (correlations of variables with themselves) are always equal to 1.00. The correlation coefficients define a type of normalization of the covariance matrix (please see

http://mathworld.wolfram.com/CorrelationCoefficient.html, in particular equations 21 and 22 for a good explanation). Matlab users find a built in function similarly to the covariance matrix calculation.

1. Generate a family of shapes with a range of transformed shapes and apply PCA. List the eigenvalues. Please note that the number of shapes should be at least twice the number of points to ensure full rank of the covariance matrix. For transformation, apply:

   (a) random translations, report and plot eigenvalues
   (b) random scale, report and plot eigenvalues
   (c) random translations and scale, report eigenvalues

2. Add random noise to each point and repeat previous experiments. Discuss eigenvalues obtained without and with noise.

3. Synthesize the major eigenmodes of deformation (e.g. modes 1 to 3) for the latter case with noise and translated and scaled shapes by creating shapes that represent the mean shape plus/minus $s * Sqrt[eigval_i]$.

4. You might Add random noise to each node. What do you expect in regard to the resulting eigenvalues?

5. **Rotation:** Generate a set of rotated shapes (only rotation with some small noise perturbation). Apply PCA, and reconstruct the eigenmodes of variation. Discuss the eigenvalues and harmonics you get, what is your interpretation?

Report

You should write up a report summarizing your procedure and discussing your results. The report should be **written in html** and **accessible to the instructor via a web-browser**, if a web-system is not available you can create a pdf file.

- Short description of how you implemented the ASM modeling.

- Description of method for reconstruction of synthetic shapes presented the major modes of deformation.

- Application to a family of shapes of your choice (simple shape models with random transformations as discussed above.

- Plot of the eigenvalues, eventual plot of the cumulative distribution function of the eigenvalues (integration).

- Image of correlation matrix for one of your experiements (e.g. scaling or translation). Discussion of correlation matrix in terms of pairwise correlation point coordinates.

- Apply your program to a set of rotated shapes. Reconstruct eigenmodes and discuss eigenmodes and eigenvalues.

Example Correlation Matrix

The following illustrates a simple example of a family of squares scaled around the origin with noise added to each corner point. The images show the family of shapes, the correlation matrix in image form, and the correlation matrix in numeric form.
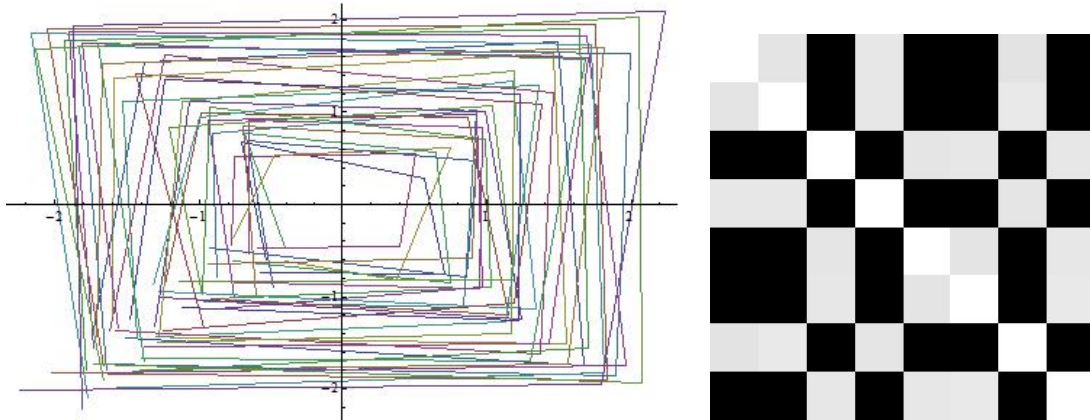

Figure 1: Left: Set of scaled shapes with random noise using 4 corner points. Right: Correlation matrix with dimension 8x8, white/grey/black maps to positive/none/negative correlation).

$$
\begin{pmatrix}
1. & 0.894096 & -0.891285 & 0.903927 & -0.902315 & -0.907884 & 0.888082 & -0.898147 \\
0.894096 & 1. & -0.896397 & 0.905367 & -0.896821 & -0.907201 & 0.910905 & -0.909616 \\
-0.891285 & -0.896397 & 1. & -0.904403 & 0.89559 & 0.904651 & -0.901327 & 0.905134 \\
0.903927 & 0.905367 & -0.904403 & 1. & -0.907947 & -0.896799 & 0.898122 & -0.913663 \\
-0.902315 & -0.896821 & 0.89559 & -0.907947 & 1. & 0.893275 & -0.894731 & 0.902413 \\
-0.907884 & -0.907201 & 0.904651 & -0.896799 & 0.893275 & 1. & -0.889672 & 0.911727 \\
0.888082 & 0.910905 & -0.901327 & 0.898122 & -0.894731 & -0.889672 & 1. & -0.904586 \\
-0.898147 & -0.909616 & 0.905134 & -0.913663 & 0.902413 & 0.911727 & -0.904586 & 1.
\end{pmatrix}
$$

Additional Exploratory Analysis: Bonus

You might apply your procedure to one or both of the following datasets for additional testing and exploration of the ASM method.

Corpus callosum shape study

The project folder includes a test set of corpus callosum structures for more testing of your module. The file includes 30 shapes with 12 points each, so that we have 24 coordinates for 30 shapes. The file is arranged as $\{\{c1,1,...,c1,24\},\{c2,1\ ,\ ...\ ,\ c2,24\},\ ...,\ \{c30,1\ ,\ ...\ ,\ c30,24\}\}$. Here, you could calculate the mean shape and the major shape variations. How many coefficients would you keep if you want to retain 90% of the energy of shape variability?
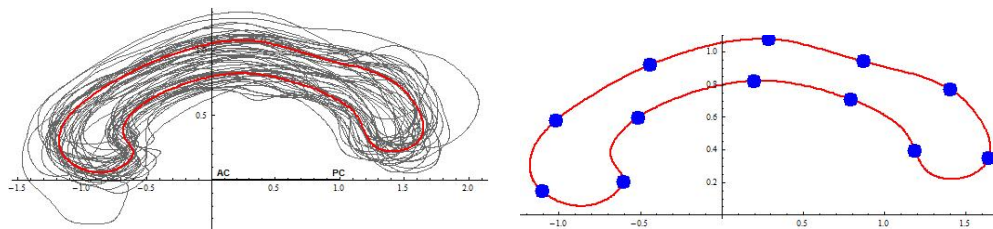

Figure 2: Left: Set of 30 corpus-callosum shapes aligned with the anatomical AC-PC coordintes. The mean shape is overlaid in red. Right: The 12 sample points overlaid for the mean shape.

Hand Dataset

Mikkel Stegman and David Delgado Gomez, DIKU, provide a statistical hand dataset (see document $http : //www2.imm.dtu.dk/pubdb/views/edoc_download.php/403/pdf/imm403.pdf$). Data has been downloaded from $http : //www2.imm.dtu.dk/\ aam/$ where you can find other interesting material.

This dataset includes 40 hands with 56 contour points each (112 x and y coordinates). The contours are not aligned (translation, rotation, scaling) but nevertheless provide interesting eigenmodes if processed as raw contours.

The formatting is as follows: Following the autor's explaination: *The file handshapes.txt contains the subset of hands used for the note alls formatet as xxxyyy, each is 56 points i.e. 112 rows and one column per shape, 40 shapes..* The string thus includes 4480 floating point coordinates. The first 2240 numbers are the x coordinates, the second half the corresponding y coordinates. The first 40 numbers represent the x-coordinates for all shapes for the first landmark, the second 40 for the second landmark until landmark 56. The datastructure is thus $\{x_{ij}, (i, 1, 40), (j, 1, 56)\}$ and followed by the same for the y-coordinates.
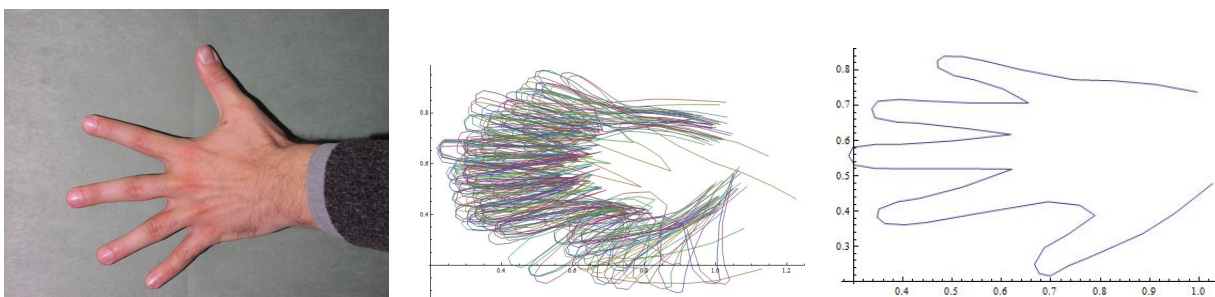

Figure 3: Left: Sample image of hand database. Middle: Set of 40 handshapes with 56 contour points each. Right: Mean shape.