BIOEN 6330 Principles of Magnetic Resonance Imaging
Homework #2
Due Tuesday, February 9, 2010

Textbook reference: Chapters 2 – 4.
Show all work and attach MATLAB code used.

1. Compute the proton magnetization density (i.e., M_o for 1 ml) at 3.0 T for a tissue (in a live person) that is 80% water.

2. Derive the equilibrium magnetization for N spin-3/2 nuclei placed inside a magnetic field B_0. Do *not* simply plug numbers in the general formula.

3. (a) Derive the expression for the steady-state longitudinal magnetization M_z^{ss} (ignoring T_2 effects) for a "train" of α pulses as a function of TR and T_1.

   (b) For given TR and T_1, what α should be used to maximize the *observed* signal?

   (c) It takes several RF pulses before the steady state condition is achieved. However, variable flip angles can be used to obtain constant signal. Suppose a TR of 0.5 sec is used for a tissue having T_1 of 1.0 sec, compute the flip angles for the first 3 RF pulses that would yield the same *observed* signal as in the steady state condition.

4. In a spin-echo T_2 measurement experiment, the following *magnitude* signals (arbitrary units) were observed for two tissues:

TE (msec)	10	20	30	60	120
tissue 1	467	351	197	67.9	3.24
tissue 2	1213	1058	974	641	354

   (a) Use an appropriate curve-fitting technique to estimate the M_o and T_2 of each tissue. Plot the signal against *TE*, showing both the measurement points and fitted results.

   (b) Solve analytically for the *TE* to obtain maximum contrast between the two tissues.

5. In a *separate* inversion-recovery T_1 measurement experiment, the following *magnitude* signals (arbitrary units) were observed for two tissues:

TI (msec)	100	200	400	800	1600	3200
tissue 1	889	684	461	99.4	385	780
tissue 2	261	217	108	118	254	339

   (a) Use an appropriate curve-fitting technique to estimate the M_o and T_1 of each tissue. Plot the signal against *TI*, showing both the measurement points and fitted results.

   (b) Solve analytically for the inversion times that can be used to eliminate (or "null") the signal from each tissue.

Extra Credit (10 points, all 3 parts required to qualify):

(a) Transform the Bloch equations into the rotating reference frame that rotates *clockwise* about the z-axis at a frequency ω, writing the rate equations separately for M_x, M_y, and M_z. (Hint: Refer to Section 3.1 of the text.)

(b) Solve numerically and plot (e.g., use MATLAB's ODE solver and `plot3` commands) the rotating-frame trajectory of the magnetization vector with initial condition $\vec{M} = M_0\vec{z}$ subject to B_0 of 1.5 T and the excitation pulse $\vec{B}_1(t) = B_1 \cos\omega t\vec{x} - B_1 \sin\omega t\vec{y}$, where B_1= 0.05 G, $0 \leq t \leq 1.0$ ms, and ω is on resonance.

(c) Repeat part (b) for the case when ω is 1.0 kHz off-resonance.