BIOEN 6330 Principles of Magnetic Resonance Imaging
Homework #3
Due Tuesday, February 23, 2010

Textbook readings: Chapters 8 and 9.

Show all work and state all assumptions.

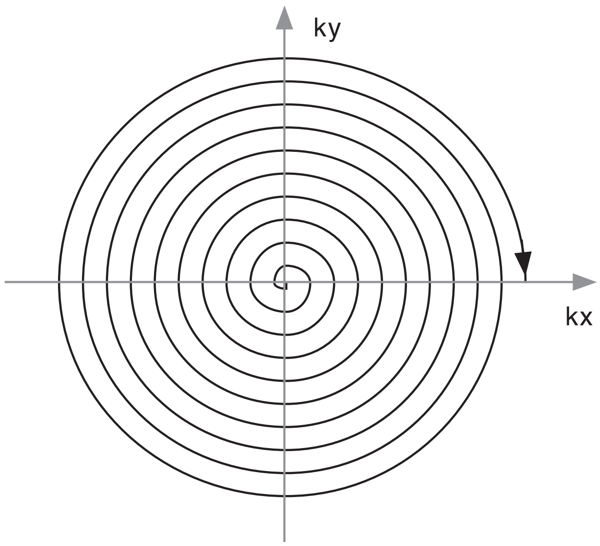
1. Recall that in linear systems theory, multiplication in time is equivalent to convolution in frequency:
   (a) Derive the expression for the *magnitude* of the Fourier spectrum of a sinusoidal signal oscillating at frequency $|\omega_0| = \gamma B_0$ subject to T2 (or T2*) decay.
   (b) Derive the expression relating T2 and the "linewidth" (use full-width-at-half-maximum) of the spectrum. What can be said about the widths of signals of fast and slow T2 decay?

2. When a surface RF coil is used in a MRI experiment, the generated B1 field drops off rather dramatically as distance from the coil, resulting in less than the prescribed flip angle being induced. To determine the effect of non-ideal angles, derive the expression for the amplitude of the spin echo generated by $\alpha_x - \tau - 2\alpha_y$ sequence.

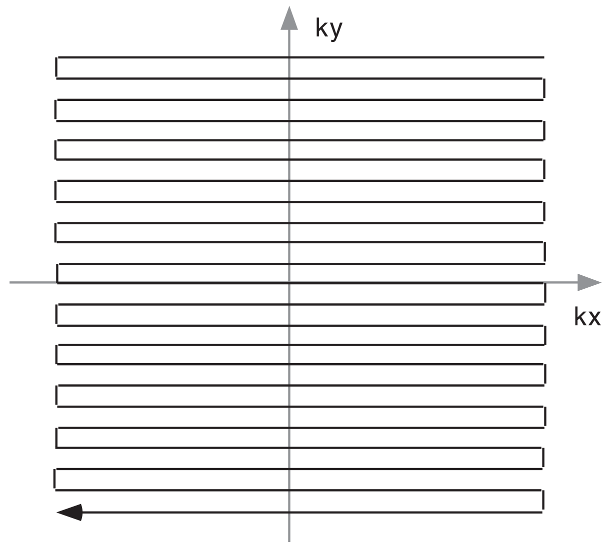
3. Suppose a uniform sample has an infinite T_1 but a finite T_2 (i.e., consider only T2 relaxation, and no "contamination" from recovering longitudinal magnetization):
   (a) For the combination of RF pulses $90°_x–\tau–180°_y–2\tau–180°_y–2\tau–180°_y$, determine the echo times and amplitudes (including polarity) for all primary spin echoes and stimulated echoes.
   (b) Repeat for the $90°_x–\tau–180°_{-y}–2\tau–180°_y–2\tau–180°_{-y}$ combination.
   (c) If signal averaging is to be performed, suggest how answers for parts (a) and (b) may be summed (or subtracted) such that some (if not all) stimulated echoes are cancelled while the spin echoes are preserved at each echo time. This is an example of the so-called "phase cycling".

4. Sketch the frequency and phase-encoding gradient pulse waveforms necessary to generate the following "single-shot" k-space trajectory. Parts (A) and (B) are used in the so-called spiral and echo-planar imaging, respectively.

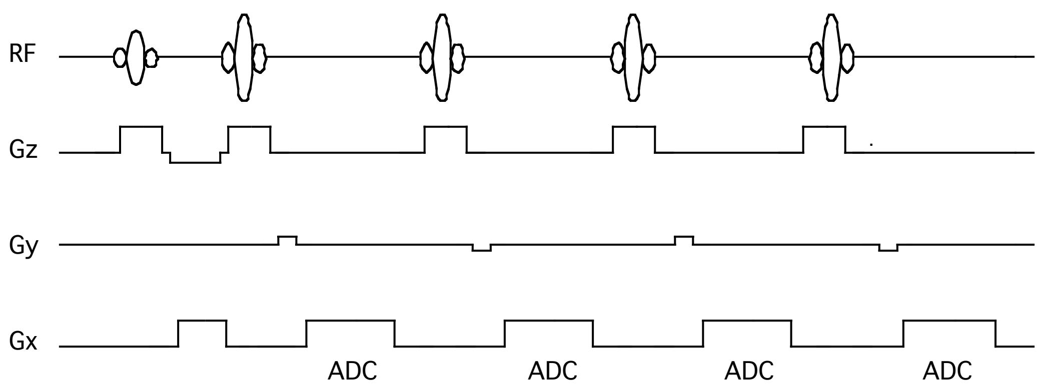
(A)



(B)



5. A fast spin-echo (FSE or RARE) sequence employs a series of 180° RF refocusing pulses to generate multiple echoes from each excitation. Consider the diagram below showing a single repetition of a sequence that acquires 4 echoes for each excitation (i.e., ETL of 4).



(a) Sketch the corresponding k-space trajectory, using "×" to mark the sampling points. Be sure to label the order of the sampling.

(b) Determine (i.e., propose and show validation) a way that only the first spin echo and its subsequent secondary echoes are acquired, and all other echoes are eliminated.