CS 6220 Spring 2011
Homework Set One
LLS Problems

Linear Least Squares

Obtain exact solution $x = x^* = [x_1^*, x_2^*, x_3^*, x_4^*]^T$ to the LLS problem

$$|| \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & -\epsilon & 0 \\ \epsilon & 0 & 0 & 0 \\ 0 & 0 & 0 & -\epsilon \\ 0 & \epsilon & 0 & 0 \\ -\epsilon & 0 & 0 & 0 \\ 0 & -\epsilon & 0 & 0 \\ 0 & 0 & 0 & \epsilon \\ 0 & 0 & \epsilon & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} ||_2 \rightarrow \min \qquad (1)$$

as a function of ϵ. There are several ways to obtain solution vector $x^* = x^*(\epsilon)$. The following steps outline one such approach.

1. Find function $f(x) = f(x_1, x_2, x_3, x_4)$ such that x which minimizes $f(x)$ is solution x^* to (1).
2. Argue that the components of x^* are equal.
3. Using 1., 2. and calculus, obtain x^*.

Next numerically solve this LLS by solving the Normal Equations for (1) via (a) Cholesky's algorithm and (b) Householder's QR. For each method experiment with the value of ϵ to see how small you can take it and still obtain an accurate solution. Hint: Pay attention to values $\epsilon \approx \sqrt{2^{-t}}$ and $\epsilon \approx 2^{-t}$.

Include a table which clearly displays the results of this experimentation as a function of ϵ. As per usual, discussions/explanations of your results will be welcome features of a successful submission.

How is it possible for LLS problem $Ax = b$ that $m \times n$ matrix A on a computer can have rank n even though on a computer $A^T A$ has rank 1?