CS 6220 Spring 2011
Homework Set Two
Eigenvalue Problems

Define column vector $u \in \mathbb{R}^n$ with i^{th} component $u_i = (-1)^{i+1}i$, $i = 1, 2, \ldots, n$. Next, normalize v to unit vector $w = u/||u||_2$. Let R be the matrix such that for each $x \in \mathbb{R}^n$ vector Rx is the reflection of x with respect to the $n-1$ dimensional hyperplane perpendicular to w. Now define matrix $A = RDR^{-1}$, where D is the $n \times n$ diagonal matrix having diagonal elements $D_{i,i} = u_i$. Let λ_{min} and λ_{max} be the smallest and largest in magnitude eigenvalues of A. Let λ^* be the eigenvalue closest to $\lambda_{max}/2$.

(i) Implement the Power and Inverse Power Methods, and test them by finding λ_{min}, λ_{max} and associated eigenvectors for n equal to 9, 51 and 99. For $n = 9$ list all the eigenvector components, while for other n list just the first and last components.

(ii) Test your Inverse Power Method with shift by applying it, for $n = 9, 51, 99$, to $(A - \sigma I)^{-1}$ with shift $\sigma = \lambda_{max}/2$ computed by using the value of λ_{max} you obtained in (i). When complete you will have numerically computed λ^*. For the case $n = 9$ what did you obtain for the corresponding eigenvector?

Work in single precision. Make known the number iterations required in each instance to achieve 8 place accuracy in your calculations of the eigenvalues. Also make known error bounds $||Av - \lambda v||_2$ for all computed eigenpairs (λ, v).

Note: The Inverse Power Method will require "inversion" of the matrix $A - \sigma I$. Look for an algorithm which has linear running time, i.e which is $O(n)$.

(iii) For any dimension n, what are the eigenvalues and eigenvectors of matrix R?