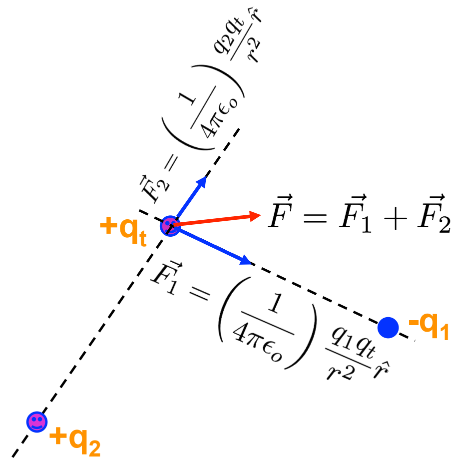
Physical/Mathematical Background

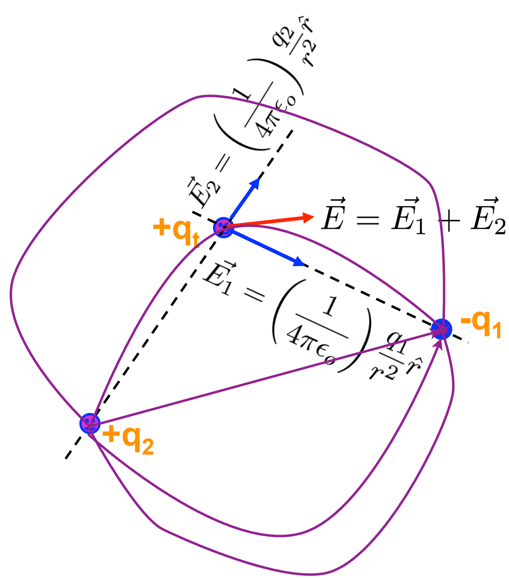
The Basics

- Fields, Flows, and Circuits
  - Electric field, potential field
  - current, conductors
  - Ohms law, IV curves, dynamic circuit analysis
- Sources, Sinks, and Vector Calculus
  - Current monopoles, dipoles
  - Volume conductor fields
  - Div, grad, curl and all that

Coulomb's Law



Superposition of Electric Field



Current and Ohm's Law

- Different forms of Ohm's Law

$$V = IR \qquad I = \frac{V}{R} \qquad I = \frac{\Delta V}{R} \qquad I = G\,\Delta V$$

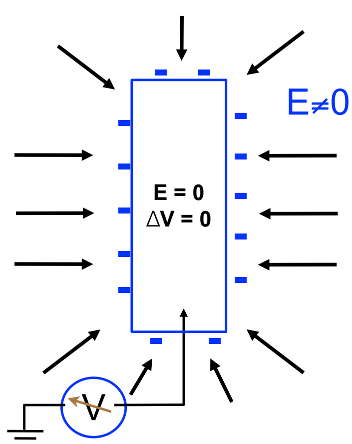
- Without potential difference there is no current!
- Essentials of Ohm's Law:
  – linear relationship between current and voltage
  – not universal, especially not in living systems

What do we mean by non-linearity in this context?

Conductors & Resistors

- Conductors
  – Electrons free to move
  – Current flow in response to electric field
  – In static state, no net charge (E=0)
- Resistors
  – Electrons less free to move
  – Create potential differences
  – Depend on material properties

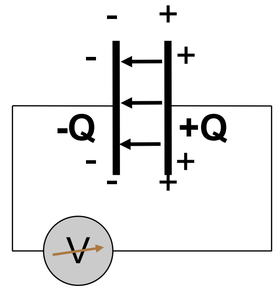
$$R = \frac{\rho l}{A}$$



Note: Electric field is the (negative) gradient of potential, E = -ΔV

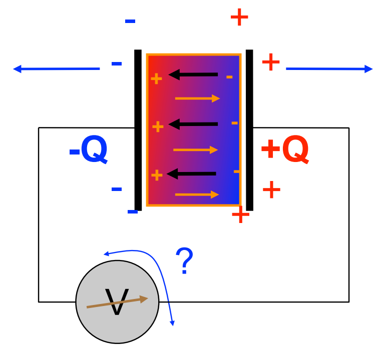
Capacitance

$$C = \frac{Q}{V} \qquad \frac{dV}{dt} = \frac{I_c}{C}$$



Total charge is a function of electric field, which depends on V and spacing.

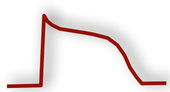
- Dielectric
  - Charges not free to move, just shift
  - E≠0 inside, opposes applied E
  - Result is increased Q for the same v and increased C



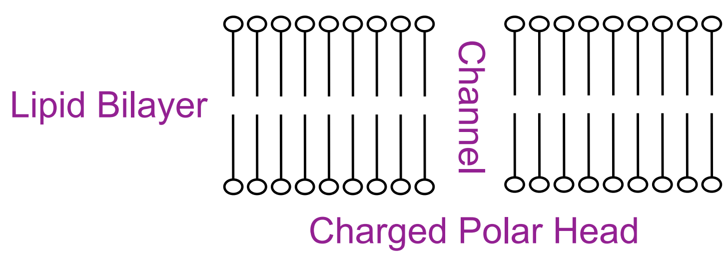
Does anything change when the plates move?

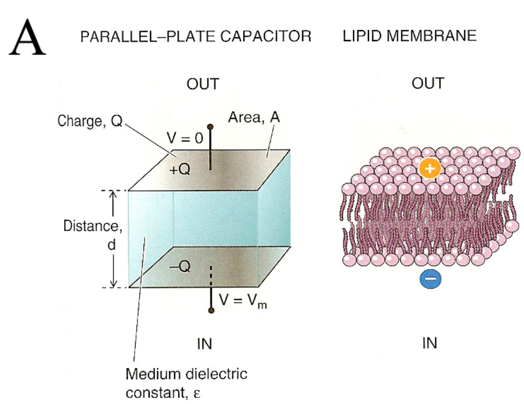
Q = CV

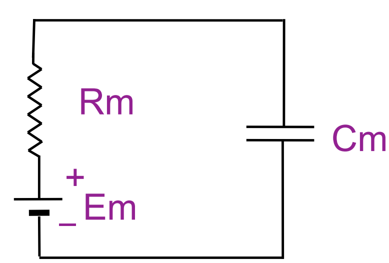
Increasing spacing decreases the E field, which means less charge and reduced capacitance



Membrane Equivalent Circuit



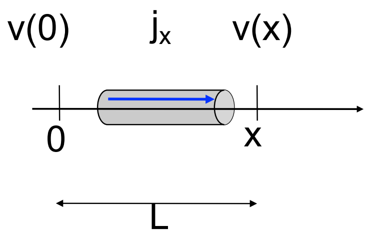




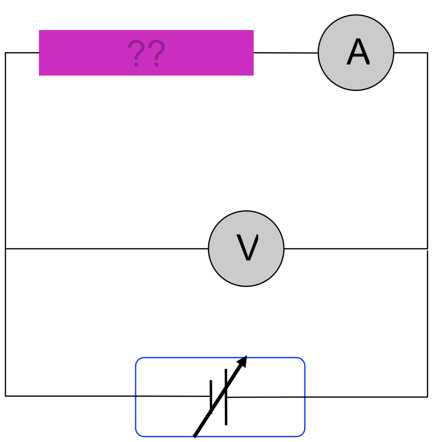
Current and Ohm's Law

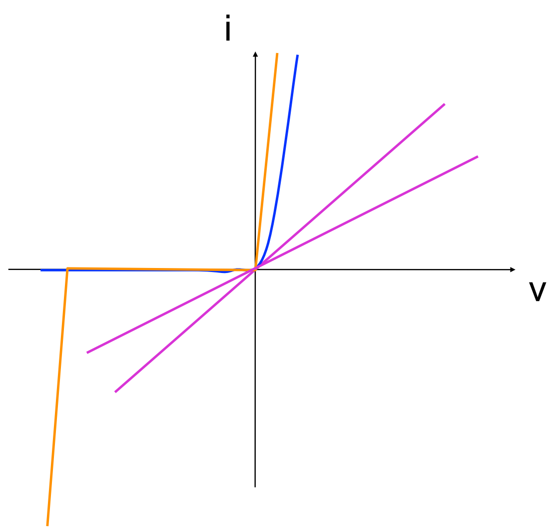
- Without potential difference there is no current!
- Without conductance, there is no current.
- Ohm's law:
  - linear relationship between current and voltage
  - not universal, especially not in living systems

$$I = \frac{1}{R} V = GV$$

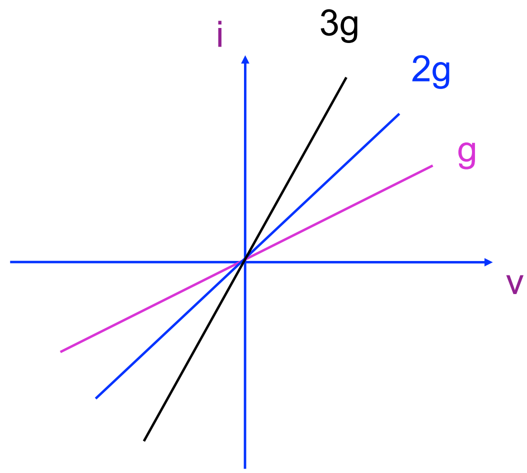


Current-Voltage (I-V)

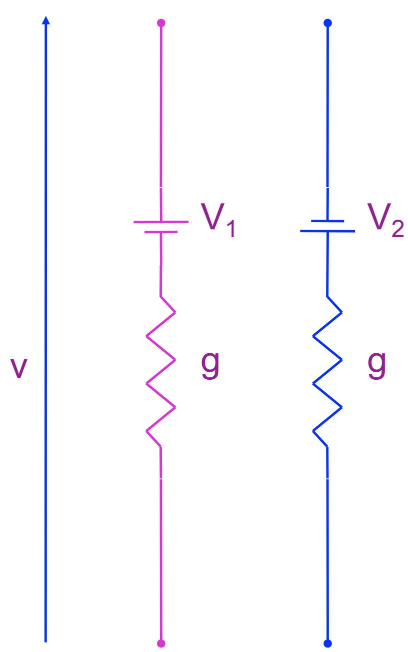


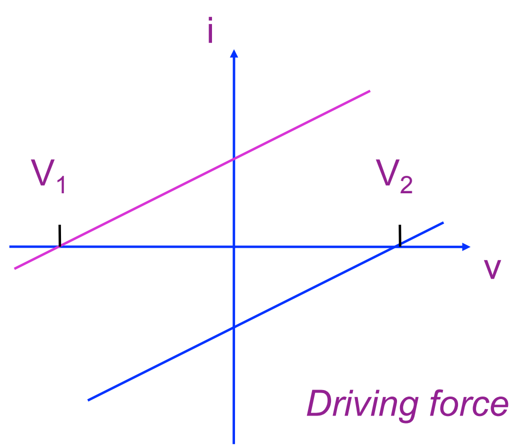


Equivalent circuits 1



Equivalent circuits 2

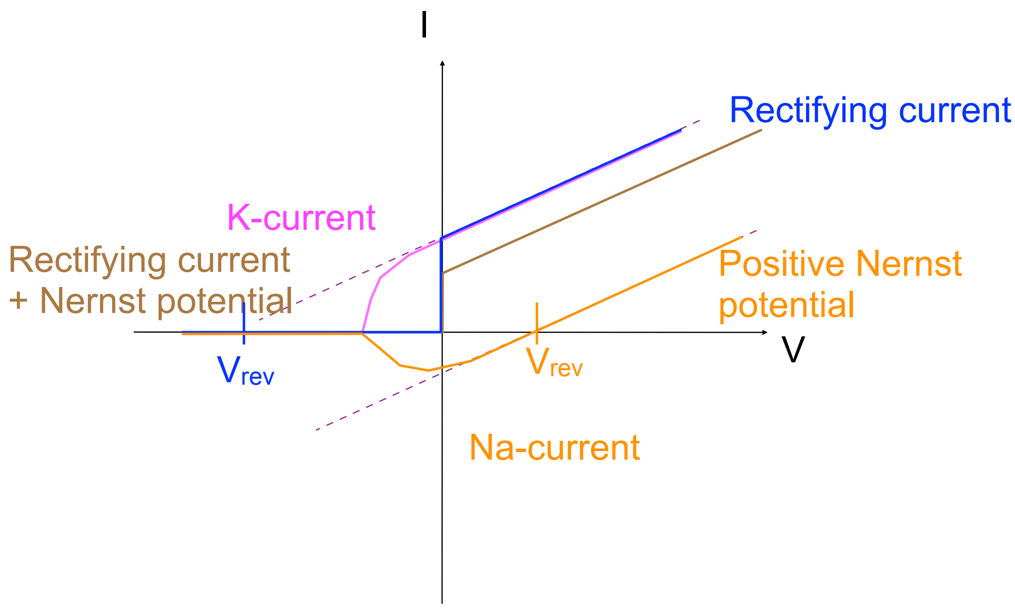




$I_1=(v+V_1)\ g$

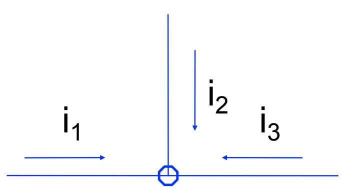
$I_2=(v-V_2)\ g$

I-V Curve Examples

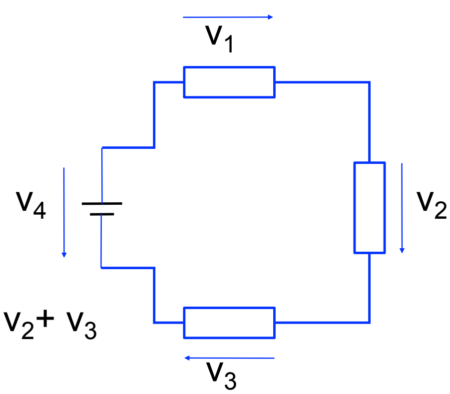


Circuit Analysis

- Conservation of charge: currents sum at nodes
- Conservation of energy: sum of voltages = 0



$i_1 + i_2 + i_3 = 0$



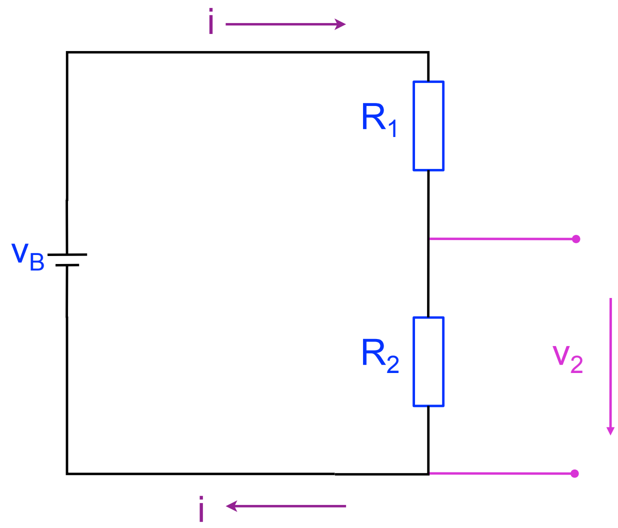
$v_4 = v_1 + v_2 + v_3$

Voltage Divider

$i = v_B/(R_1+R_2)$

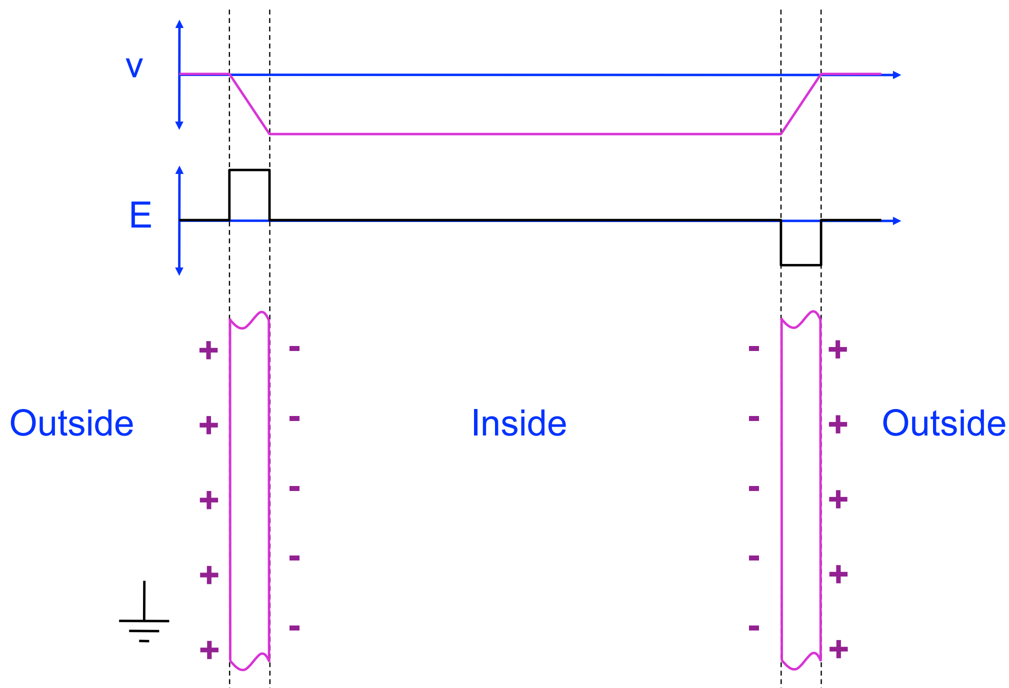
$i = v_2/R_2$

$v_2 = v_B\, R_2/(R_1+R_2)$



Examples of voltage dividers in EP measurements?

Electrical Profile of a Cell

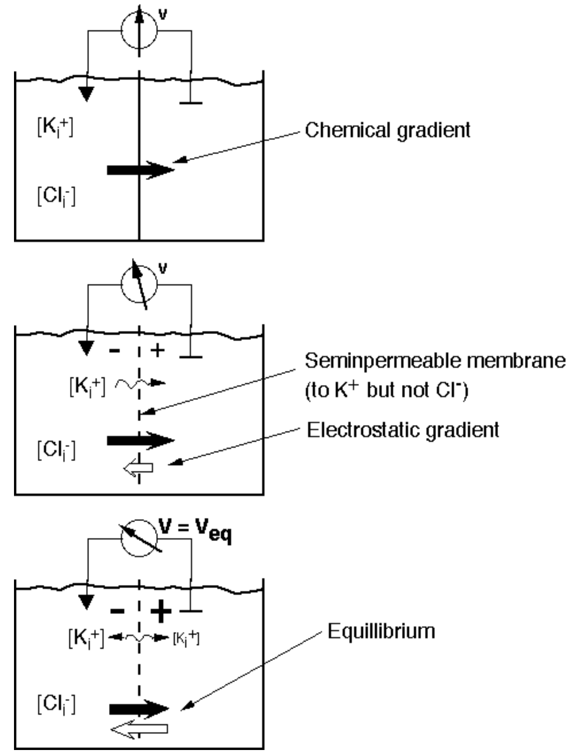


Equilibrium Potential

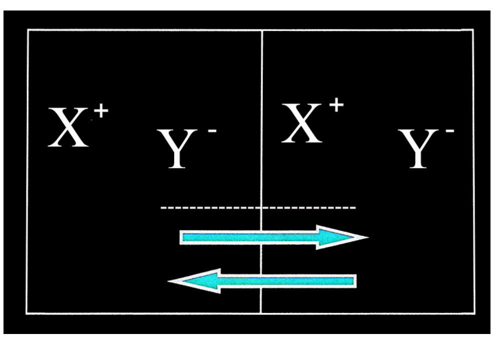
a) Membrane is impermeable

b) Membrane becomes permeable to potassium only (semipermeable)

c) Equilibrium established when electrostatic and chemical gradients balance.



Nernst Equillibrium



Electrical and Chemical work

$$\Delta W_e = \Delta N \cdot zFV$$

$$\Delta W_c = \Delta N \cdot RT \log_e \frac{[X]_1}{[X]_2}$$

$$\Delta N \cdot zFV = \Delta N \cdot RT \log_e \frac{[X]_1}{[X]_2}$$

At equilibrium there will be no net movement of X

$$V_e = \frac{RT}{zF} \log_e \frac{[X]_1}{[X]_2}$$

Example Nernst Potentials

$$E = \frac{25}{z} \log_e \frac{[X]_1}{[X]_2}$$

or $$E = \frac{58}{z} \log_{10} \frac{[X]_1}{[X]_2}$$

	Ion	External	Internal	Nernst Potential (mV)
Frog muscle	K	2.25	124	-101
	Na	109	10.4	+59
	Cl	77.5	1.5	-99
Squid axon	K	20	400	-75
	Na	440	50	+55
	Cl	560	108	-41

Diffusion

- Driven by thermodynamic random motion
- Similar linear relationship as Ohm's Law

$$D = \Delta C P$$

- where D = rate of diffusion
- C = concentration
- P = permeability