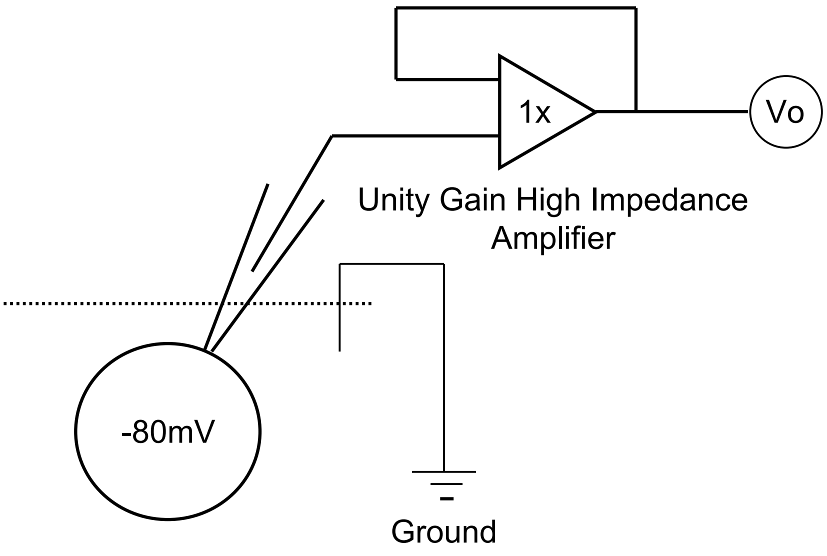
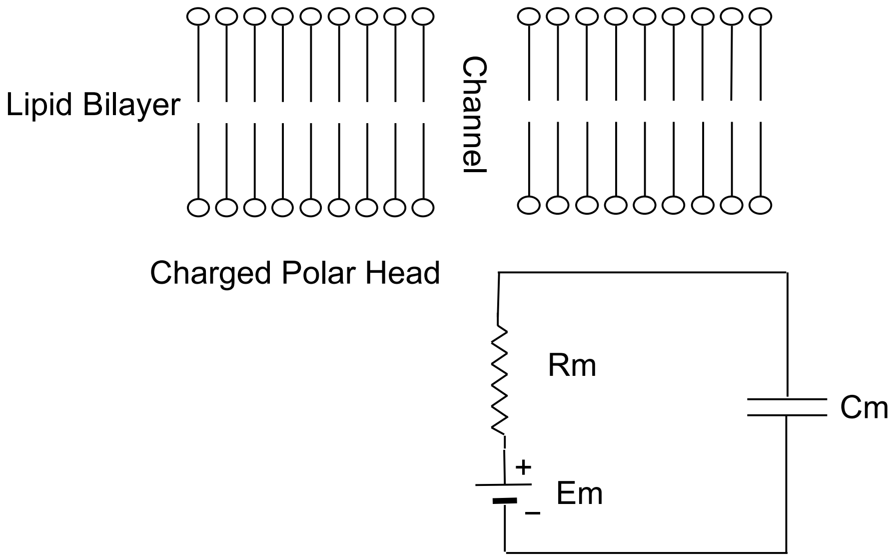
## Ionic concentration (mM)

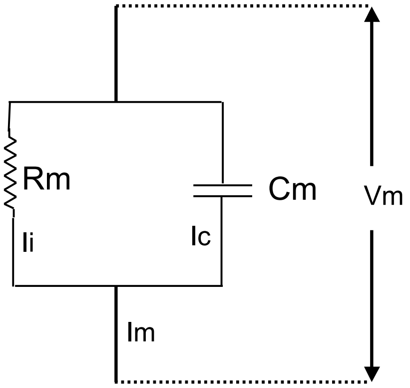
| | Ion | External | Internal | Nernst Potential (mV) |
|---|---|---|---|---|
| Frog muscle | K | 2.25 | 124 | -101 |
| | Na | 109 | 10.4 | +59 |
| | Cl | 77.5 | 1.5 | -99 |
| Squid axon | K | 20 | 400 | -75 |
| | Na | 440 | 50 | +55 |
| | Cl | 560 | 108 | -41 |

## Measuring Membrane Potential



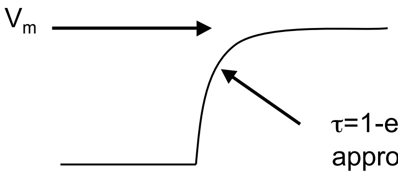
## Membrane Equivalent Circuit





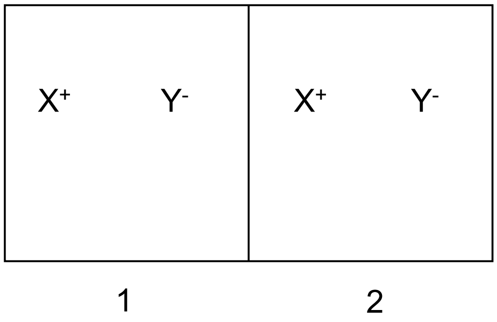
$Im=Ic+Ii$
By Ohms Law $Im=V/Rm$
and $Im=Cm.dV/dt + Im$

The solution to this equation is $V=ImRm(1-e^{t/\tau})$
$t/\tau$ is the membrane time constant=$RmCm$

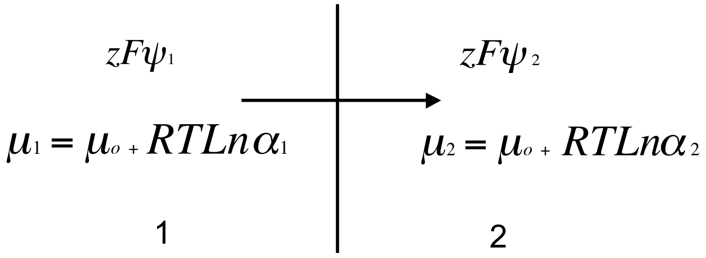


$\tau = 1 - e^{-1}$ time required to reach approx 2/3 of Vm after a step change in current.

## Concentration Cells



## *Nernst Equation*



$zF\psi_1$

$zF\psi_2$

$$\mu_1 = \mu_o + RTLn\alpha_1$$

$$\mu_2 = \mu_o + RTLn\alpha_2$$

1

2

$$C = \gamma\alpha$$

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$$\mu_1 = \mu_o + RTLn\alpha_1 + zF\psi_1$$

$$\mu_2 = \mu_o + RTLn\alpha_2 + zF\psi_2$$

*At equilibrium* $\mu_1=\mu_2$

*We can define the electrochemical potential as:*

$$\mu_1 = \mu_o + RTLn\alpha_1 + zF\psi_1$$

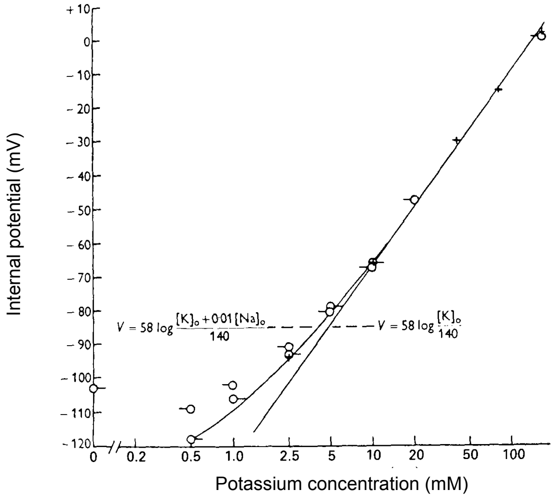
$$\mu_2 = \mu_o + RTLn\alpha_2 + zF\psi_2$$

*At equilibrium* $\mu_1 = \mu_2$

$$\therefore RTLn\alpha_1 + zF\psi_1 = RTLn\alpha_2 + zF\psi_2$$

$$\psi_1 - \psi_2 = E = \frac{RT}{zF} Ln \frac{\alpha_1}{\alpha_2} = \frac{RT}{zF} Ln \frac{C_1}{C_2}$$

$$E = \frac{RT}{zF} Ln \frac{C_1}{C_2}$$ *Nernst equation*



The effect of the external potassium ion concentration on the membrane potential of isolated frog muscle fibres. The external solutions were chloride-free, the principle anion being sulphate. (From Hodgkin and Horowicz, 1959.)

## Hodgkin Goldman Katz Equation (Modified).

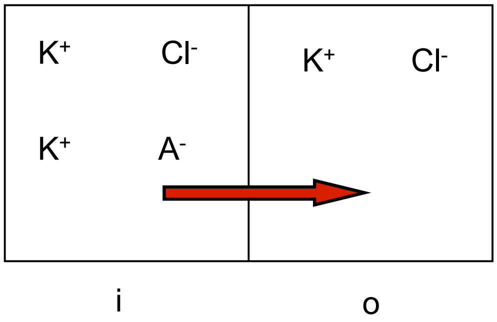
It is possible to derive an eqution that predicts the Effect on Em of permeabilites to other ion e.g Na.

For the contribution of Na to membrane potential We may write the following equation:

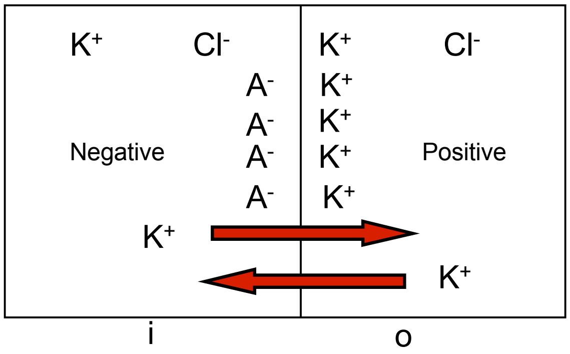
$$E=\frac{RT}{F}\ln\frac{[K]_o+\alpha[Na]_o}{[K]_i+\alpha[Na]_i}$$

$\alpha=P_{Na}/P_K$ Assume $\alpha=0.01$

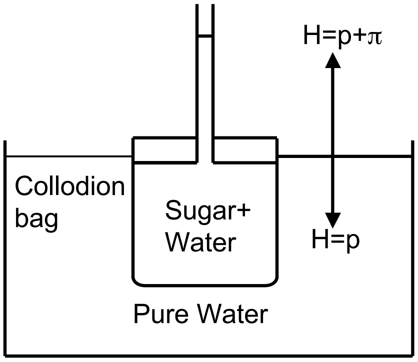
## How Does the Membrane Potential Arise Donnan's Theory.



## Donnan Equilibrium

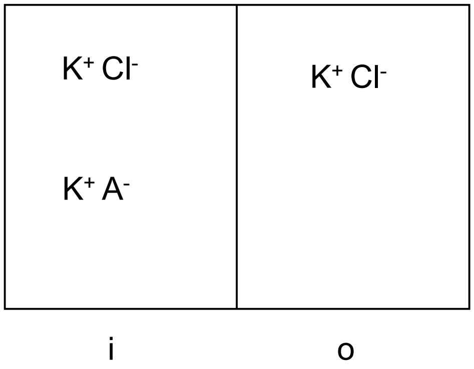


## Osmosis



Add a small quantity of KA to i. The membrane is impermeant to A
K is at higher concentration in I and diffuses to o. Cl follows the K.
eventually and equibrium arises in which $[K]_I$ is not equal to $[K]_o$ and
$[Cl]_I$ is not equal to $[Cl]_{o.}$ This creates a concentration cell and the condition
$E_{k=}E_{cl}$
must hold.

The Donnan equilibrium system



$K^+$ $Cl^-$

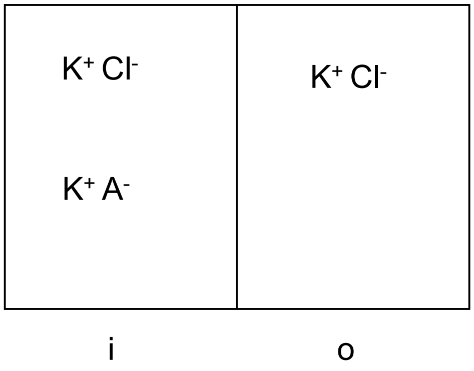
$K^+$ $A^-$

$K^+$ $Cl^-$

i

o

## The Donnan equilibrium system



$K^+$ $Cl^-$

$K^+$ $A^-$

$K^+$ $Cl^-$

i

o

## The Donnan Product

For $K^+$

$$E_k = \frac{RT}{zF} \ln \frac{[K+]_o}{[K+]_i}$$

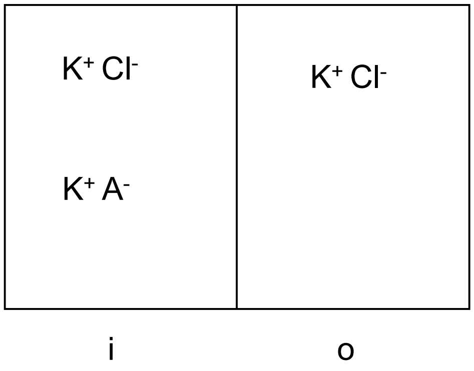
For $Cl^-$

$$E_{Cl} = \frac{RT}{-zF} \ln \frac{[Cl-]_o}{[Cl-]_i}$$

$$\therefore E_K = E_{Cl}$$

$$\therefore \frac{[K]_o}{[K]_i} = \frac{[Cl]_o}{[Cl]_i} = [K]_i \times [Cl]_i = [K]_o \times [Cl]_o \quad \text{(Donnan Product)}$$

## The Donnan equilibrium system



$K^+ Cl^-$

$K^+ A^-$

$K^+ Cl^-$

i

o

## The Osmotic Argument

We cannot apply this simple Donnan system to an animal cell because it will swell.

For approximate electrical neutrality $[K_o]=[Cl_i]$ and $[K_o]>[Cl_i]$.

now $[K]_i x[Cl]_i = [K]_o x[Cl]_o$,

therefore

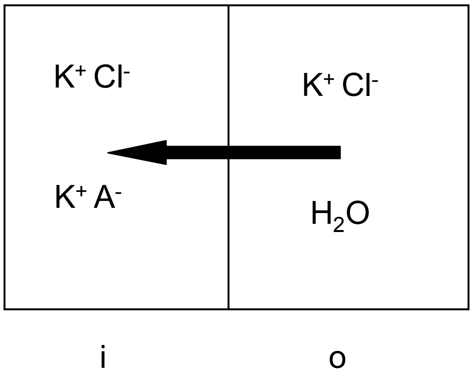
$K]_i x[Cl]_i > [K]_o x[Cl]_o$

therefore

$K]_I+[Cl]_I+[A] > [K]_o+[Cl]_o$

therefore the osmotic concentration is greater in i than in o. Thus if the constant volume constraint is moved water moves from i to o.

## The Donnan equilibrium system

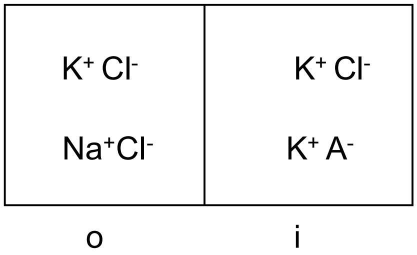


$K^+ Cl^-$ $K^+ Cl^-$

$K^+ A^-$ $H_2O$

i o

## Impermeant Anion in Outer Compartment prevents Water Loss



$K^+ Cl^-$ $K^+ Cl^-$

$Na^+Cl^-$ $K^+ A^-$

o i

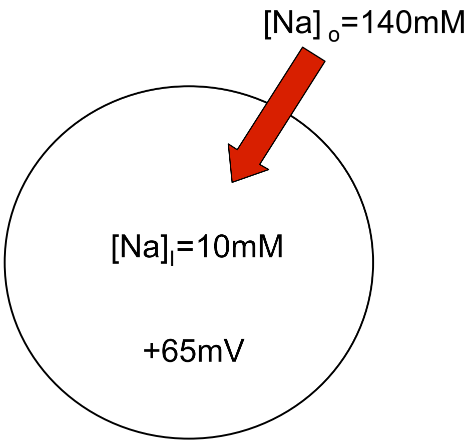
With and impermeant anion present water will move untill the Donnan equilibrium is established i.e. $[K]_i x[Cl]_i = [K]_o x[Cl]_o$,

## Driving Forces

Ohms Law I=gV

$$E_{Na}=\frac{RT}{zF}\ln\frac{[Na+]_o}{[Na+]_o}$$

$E_{Na}$=+65mV

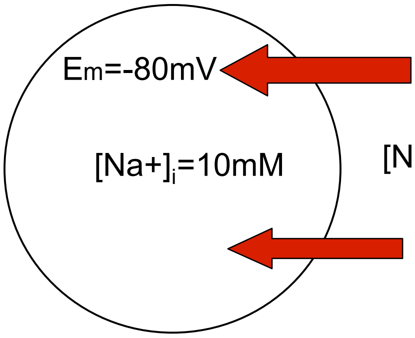


$[Na]_o$=140mM

$[Na]_I$=10mM

+65mV

65mV required to oppose Na diffusion. -65mV=effective force exerted by the Na gradient.Thus the chemical driving force is $-E_{Na}$



$E_m$=-80mV

Electrical Driving Force =$E_m$ (mV)

$[Na+]_i$=10mM

$[Na^+]_o$=140mM

Force Driving Diffusion=$-E_{Na}$

Net Driving Force(mV)=$E_m+(-E_{Na})$
Or $E_m-E_{Na}$ (mV)

## Ohms Law and Electrophysiolgy

Net Driving Force(mV)=$E_m-E_{Na}$ (mV)

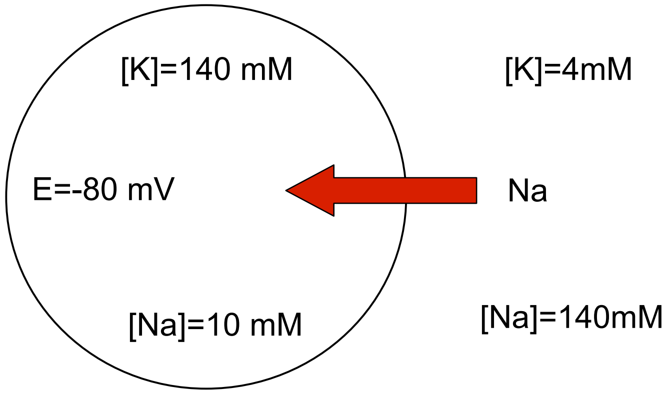
From Ohms Law Na current ,$I_{Na}$ (ion flux) will be given by:

$$I_{Na}=g_{Na}(E_m-E_{Na})$$

When $E_m=E_{Na}$ $I_{Na}=0$

By changing $E_m$ until $I_{Na}=0$ we can find $E_{Na}$.

Calculations with the Nernst equation indicate that $[Na_i]$ should = 3,434 mM if it is at electrochemical equilibrium.



[K]=140 mM

[K]=4mM

E=-80 mV

Na

[Na]=10 mM

[Na]=140mM

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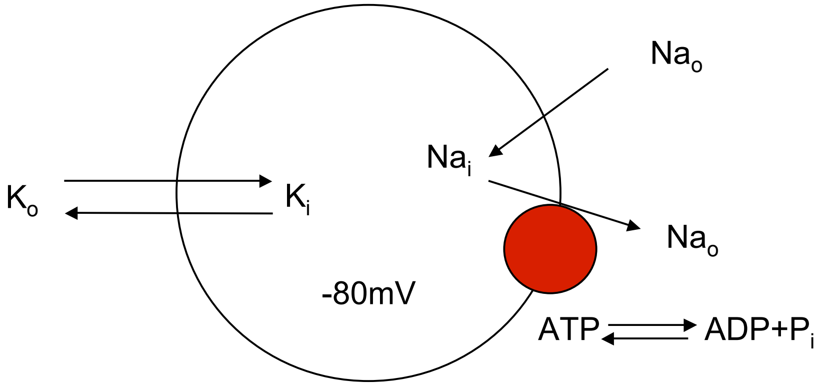
Na Pump Maintains the Inward Na Gradient



$Na_o$

$Na_i$

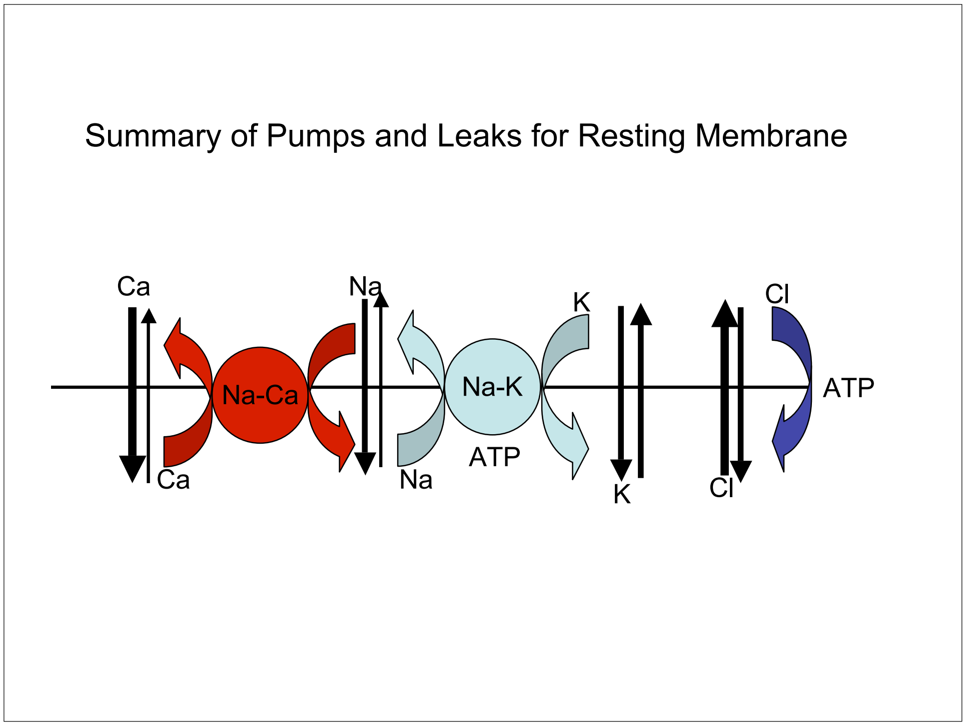
$K_o$

$K_i$

$Na_o$

-80mV

$ATP \rightleftarrows ADP+P_i$



Summary of Pumps and Leaks for Resting Membrane