

closer look at recursion, why it's important to Haskell and how we can work out ...

#### Recursion

pages.cs.wisc.edu/.../6.RECURSION.ht... 
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The original call causes 2 to be output, and then a recursive call is made, creating a
clone with k == 1. That clone executes line 1: the if condition is false; line 4: ...

# RECURSION

cs2420 | Introduction to Algorithms and Data Structures | Spring 2015

## administrivia...

-assignment 4 due on Thursday at midnight

-a personal testimony...

-no change of due dates for homework

-midterm next Tuesday

## last time...

# selection vs insertion

WORST: AVERAGE: BEST: O(N<sup>2</sup>) O(N<sup>2</sup>) O(N<sup>2</sup>) O(N<sup>2</sup>) O(N<sup>2</sup>) O(N)

WHICH ONE PERFORMS BETTER IN PRACTICE? A) selection B) insertion

# what we want...

-a sorting algorithm that has subquadratic complexity

-swapping adjacent items removes exactly 1 inversion

SWAP REMOVES 1 INVERSION

-what if we consider swapping nonadjacent pairs?

-removes inversions not involved with the swap





1) set the gap size to N/2

2) consider the subarrays with elements at **gap size** from each other

3) do insertion sort on each of the subarrays

4) divide the **gap size** by 2

5) repeat steps 2 - 4 until the is **gap size** is <1

## WHAT DOES THIS LOOK LIKE?

#### HOW DO WE DESCRIBE INSERTION SORT WITH RESPECT TO SHELLSORT?

```
DIMINISHING GAP SEQUENCE.
void shellSort(int[] arr)
  for (gap = arr.length/2; gap > 0; gap /= 2)
    for(i = gap; i < arr.length; i++)</pre>
      val = arr[i]; ----- ITEM TO BE INSERTED
      -for(j = i-gap; j >= 0 && arr[j] > val; j -= gap)
         arr[j+gap] = arr[j];
      arr[j+gap] = val; ---- INSERT ITEM
```



-what is recursion? and some examples...

-driver methods

-the overhead of recursion

### re · cur · sion

#### [ri-**kur**-zh*uh* n] *noun*

see recursion.

-recursion is a problem solving technique in which the solution is defined in terms of a simpler (or smaller) version of the problem
 -break the problem into smaller parts
 -solve the smaller problems
 -combine the results

-a recursive method calls itself

-some functions are easiest to define recursively sum(N) = sum(N-1) + N

-there must be at least on *base case* that can be computed without recursion

-any recursive call must make progress towards the base case!

## a simple example sum(N) = sum(N-1) + N



HOW CAN WE SOLVE THE SAME PROBLEM WITHOUT RECURSION? WHICH IS BETTER, THE RECURSIVE SOLUTION OR THE ALTERNATIVE?

#### -how to compute N! N! = N \* N-1 \* N-2 \* ... \* 2 \* 1

-how would you compute this using a for-loop?

-how would you compute this using recursion? -think about:

-what is the base case?

-what is recursive?

#### -how to compute **N! N!** = **N** \* **N-1** \* **N-2** \* ... \* **2** \* **1**

-how would you compute this using a for-loop?

-how would you compute this using recursion? A) c -think about: -what is the base case? -what is recursive? A) C B) log N C) N D) N log N E) N<sup>2</sup>

## WHAT IS THE COMPLEXITY OF THE FOR-LOOP METHOD?

**N**<sup>3</sup>

#### -how to compute **N! N!** = **N** \* **N-1** \* **N-2** \* ... \* **2** \* **1**

-how would you compute this using a for-loop?

-how would you compute this using recursion? A) c -think about: -what is the base case? -what is recursive? A) C B) log N C) N D) N log N E) N<sup>2</sup>

## WHAT IS THE COMPLEXITY OF THE RECURSIVE METHOD?

**N**<sup>3</sup>

# -write a recursive method that computes A/B -do integer division -/ operator not allowed, can only use -don't worry about negative input or divide-by-zero

```
public static int divide(int a, int b)
{
    ...
}
HINT: 9/2 = 1 + (7/2)
```

-recursion often seems like

-when writing a recursive method, just assume that the function you're writing already works, so you can use it to help solve the problem

-once you've worked out the recursion, think about the base case, and you're done

## driver methods

# divide and conquer

- -divide and conquer is an important problem solving technique that makes use of recursion
  - -divide: smaller problems are solved recursively (except for base cases!)
  - -**conquer:** solutions to the subproblems form the solution to the original problem
- -typically, an algorithm containing more than one recursive call is referred to as divide and conquer
- -subproblems are usually disjoint (non-overlapping)



-write a recursive method to perform a binary search -assume an (ascending) sorted list

#### -HINT

-check if middle item is what we're looking for *-if so, return true*-else, figure out if item is the left or right half *-repeat on that half*

-base case(s)???

-recursive methods often have unusual parameters
-at the top level, we just want:
 binarySearch(arr, item);

-but in reality, we have to call:

binarySearch(arr, item, 0, arr.length-1);

-driver methods are wrappers for calling recursive methods

-driver makes the initial call to the recursive method, knowing what parameters to use

-is not recursive itself

```
public static boolean binarySearch(arr, item) {
    return binarySearchRecursive(
        arr, item, 0, arr.length-1);
```

-another useful feature of driver methods is error checking (or, validity checks)

-do the error checking *only* in the driver method, instead of redundantly doing it every time in the recursion

WHAT IS SOMETHING TO CHECK FOR IN OUR BINARY SEARCH METHOD?

public static boolean binarySearch(arr, item){
 if (arr == null) // only check this once
 return false;

```
return binarySearchRecursive(
    arr, item, 0, arr.length-1);
}
```

## overhead of recursion

# method calls

-every time a method is invoked, a unique "frame" is created
-contains local variables and state
-put on the call stack

-when that method returns, execution resumes in the calling method

-this is how methods know where to return to!



# recursive calls

-create multiple frames of the same method

-but each frame has different arguments

factorial(1)
factorial(2)
factorial(3)
factorial(4)
main

# recursion, beware

#### -do not use recursion when a simple loop will do

-growth rates may be the same, but...

-...there is a lot of overhead involved in setting up the method frame

-way more overhead than one iteration of a for-loop

-do not do redundant work in a recursive method -move validity checks to a driver method

-too many recursive calls will overflow the call stack -stack stores state from all preceding calls

## recap

# 4 recursion rules

- 1. always have at least one case that can be solved without using recursion
- 2. any recursive call must progress toward a base case
- 3. always assume that the recursive call works, and use this assumption to design your algorithms
- 4. never duplicate work by solving the same instance of a problem in separate recursive calls

## next time...

#### -reading -chapters 7 & 8.5 - 8.8

#### -homework -assignment 4 due Thursday

-(short) midterm review on Thursday