BINARY HEAP

cs2420 | Introduction to Algorithms and Data Structures | Spring 2015

administrivia…

-assignment 10 is due on Thursday

-midterm grades out tomorrow

last time…

-a **hash table** is a general storage data structure

-insertion, deletion, and look-up are all **O(c)**

-like a stack, but not limited to top item

	Access	Insertion	Deletion	Notes
Hash Table	Constant	Constant	Constant	Magic?

-underlying data structure is just an array

-requires that all data types inserted have a hash function

-map the *hash value* to a valid index of the array using %

-use hash value to instantly look-up the index of any item

- -insertion, deletion, and search: **O(1)**
  - *-assuming the hash function is* **O(1)***!*

linear probing

-remember: *it is NOT required that two non-equal object have different hash values*

-because of this, it is possible for two different objects to has to the same index

- -this is called a **collision**

INSERT:

12, 15, 17, 46, 89, 90, 92

COLLISION! WHERE CAN WE PUT 92? (→ index 2)

array:	90		12			15	46	17		89
index:	0	1	2	3	4	5	6	7	8	9

clustering

-if an item's natural spot is taken, it goes in the next open spot, making a cluster for that hash

- -**clustering** happens because once there is a collision, there is a high probability that there will be more
- -this means that any item that hashes into the cluster will require several attempts to resolve the collision

-feedback loop:

- -the bigger the clusters are, the more likely they are to be hit
- -when a cluster gets hit, it gets bigger

quadratic probing

-quadratic probing attempts to deal with the clustering problem

-if `hash(item) = H`, and the cell at `H` is occupied:

- -try $H+1^2$
- -then $H+2^2$
- -then $H+3^2$
- -and so on…
- -wrap around to beginning of array if necessary

separate chaining

-why not make each spot in the array capable of holding more than one item?

- -use an array of linked lists
- -hash function selects index into array
- -called **separate chaining**

-for insertion, append the item to the end of the list

- -insertion is **O(1)** if we have what?

-searching is a linear scan through the list

- -fast if the list is short

a bit more on hash functions…

-ints have an obvious hash value

array:	90		12			15	46	17		89
index:	0	1	2	3	4	5	6	7	8	9

-what about Strings? Books? Shapes?…

-we must not overlook the requirement of a *good* hash functions

remember...

-hash functions take any item as input and produce an integer as output

-given the same input the function always returns the same output

-two different inputs MAY have the same hash value

thinking about `chars` and `Strings`

- ASCII defines an encoding for characters
  - ‘a’ = 97
  - ‘b’ = 98
  - …
  - ‘z’ = 122
  - ‘2’ = 50
  - …

- the `char` type is actually just a small integer
  - 8 bits instead of the usual 32

Dec	Hx	Oct	Char	Dec	Hx	Oct	Html	Chr	Dec	Hx	Oct	Html	Chr	Dec	Hx	Oct	Html	Chr
0	0	000	NUL (null)	32	20	040		Space	64	40	100	@	@	96	60	140	`	`
1	1	001	SOH (start of heading)	33	21	041	!	!	65	41	101	A	A	97	61	141	a	a
2	2	002	STX (start of text)	34	22	042	"	"	66	42	102	B	B	98	62	142	b	b
3	3	003	ETX (end of text)	35	23	043	#	#	67	43	103	C	C	99	63	143	c	c
4	4	004	EOT (end of transmission)	36	24	044	$	$	68	44	104	D	D	100	64	144	d	d
5	5	005	ENQ (enquiry)	37	25	045	%	%	69	45	105	E	E	101	65	145	e	e
6	6	006	ACK (acknowledge)	38	26	046	&	&	70	46	106	F	F	102	66	146	f	f
7	7	007	BEL (bell)	39	27	047	'	'	71	47	107	G	G	103	67	147	g	g
8	8	010	BS (backspace)	40	28	050	(	(	72	48	110	H	H	104	68	150	h	h
9	9	011	TAB (horizontal tab)	41	29	051	)	)	73	49	111	I	I	105	69	151	i	i
10	A	012	LF (NL line feed, new line)	42	2A	052	*	*	74	4A	112	J	J	106	6A	152	j	j
11	B	013	VT (vertical tab)	43	2B	053	+	+	75	4B	113	K	K	107	6B	153	k	k
12	C	014	FF (NP form feed, new page)	44	2C	054	,	,	76	4C	114	L	L	108	6C	154	l	l
13	D	015	CR (carriage return)	45	2D	055	-	-	77	4D	115	M	M	109	6D	155	m	m
14	E	016	SO (shift out)	46	2E	056	.	.	78	4E	116	N	N	110	6E	156	n	n
15	F	017	SI (shift in)	47	2F	057	/	/	79	4F	117	O	O	111	6F	157	o	o
16	10	020	DLE (data link escape)	48	30	060	0	0	80	50	120	P	P	112	70	160	p	p
17	11	021	DC1 (device control 1)	49	31	061	1	1	81	51	121	Q	Q	113	71	161	q	q
18	12	022	DC2 (device control 2)	50	32	062	2	2	82	52	122	R	R	114	72	162	r	r
19	13	023	DC3 (device control 3)	51	33	063	3	3	83	53	123	S	S	115	73	163	s	s
20	14	024	DC4 (device control 4)	52	34	064	4	4	84	54	124	T	T	116	74	164	t	t
21	15	025	NAK (negative acknowledge)	53	35	065	5	5	85	55	125	U	U	117	75	165	u	u
22	16	026	SYN (synchronous idle)	54	36	066	6	6	86	56	126	V	V	118	76	166	v	v
23	17	027	ETB (end of trans. block)	55	37	067	7	7	87	57	127	W	W	119	77	167	w	w
24	18	030	CAN (cancel)	56	38	070	8	8	88	58	130	X	X	120	78	170	x	x
25	19	031	EM (end of medium)	57	39	071	9	9	89	59	131	Y	Y	121	79	171	y	y
26	1A	032	SUB (substitute)	58	3A	072	:	:	90	5A	132	Z	Z	122	7A	172	z	z
27	1B	033	ESC (escape)	59	3B	073	;	;	91	5B	133	[	[	123	7B	173	{	{
28	1C	034	FS (file separator)	60	3C	074	<	<	92	5C	134	\	\	124	7C	174			|
29	1D	035	GS (group separator)	61	3D	075	=	=	93	5D	135	]	]	125	7D	175	}	}
30	1E	036	RS (record separator)	62	3E	076	>	>	94	5E	136	^	^	126	7E	176	~	~
31	1F	037	US (unit separator)	63	3F	077	?	?	95	5F	137	_	_	127	7F	177	&#127;	DEL

-a `String` is essentially an array of `char`

```
String s = "hello";
```

104	**101**	**108**	**108**	**111**

-Java hides these details

-how can we use this to create a hash function for `Strings`?

review

- **O(1)** for all major operations
  - assuming λ is managed

- linear probing
  - has clustering problems

- quadratic probing
  - has lesser clustering problems
  - requires $\lambda < 0.5$, and prime table size

- separate chaining
  - probably the easiest to implement, as well as the best performing

WHAT IS THE LOAD FACTOR λ FOR THE FOLLOWING HASH TABLE?

A) **4**
B) **6**
C) **0.4**
D) **0.5**
E) **0.6**

104	34		19	111	98		52		

USING LINEAR PROBING, IN WHAT INDEX WILL ITEM 93 BE ADDED?

A) **1**
B) **5**
C) **6**
D) **7**

array:	49		9	58	34				18	89
index:	0	1	2	3	4	5	6	7	8	9

USING QUADRATIC PROBING, IN WHAT INDEX WILL ITEM 22 BE ADDED?

A) **1**
B) **5**
C) **6**
D) **7**

array:	49		9	58	34				18	89
index:	0	1	2	3	4	5	6	7	8	9

recap

-i heart hash tables

- -collection structure with O(1) for major operations

-but!…

- -hash function must minimize collisions
    - *-should evenly distribute values across all possible integers*
- -collisions must be carefully dealt with
- -hash function runtime must be fast

- -no ordering
    - *-how do we find the smallest item in a hash table?*
    - *-in a BST?*

priority queues

- a priority queue is a data structure in which access is limited to the minimum item in the set
  - `add`
  - `findMin`
  - `deleteMin`

- add location is unspecified, so long as the the above is always enforced

- what are our options for implementing this?

- option 1: a linked list
  - add: **O(1)**
  - findMin: **O(N)**
  - deleteMin: **O(N)** (including finding)

- option 2: a sorted linked list
  - add: **O(N)**
  - findMin: **O(1)**
  - deleteMin: **O(1)**

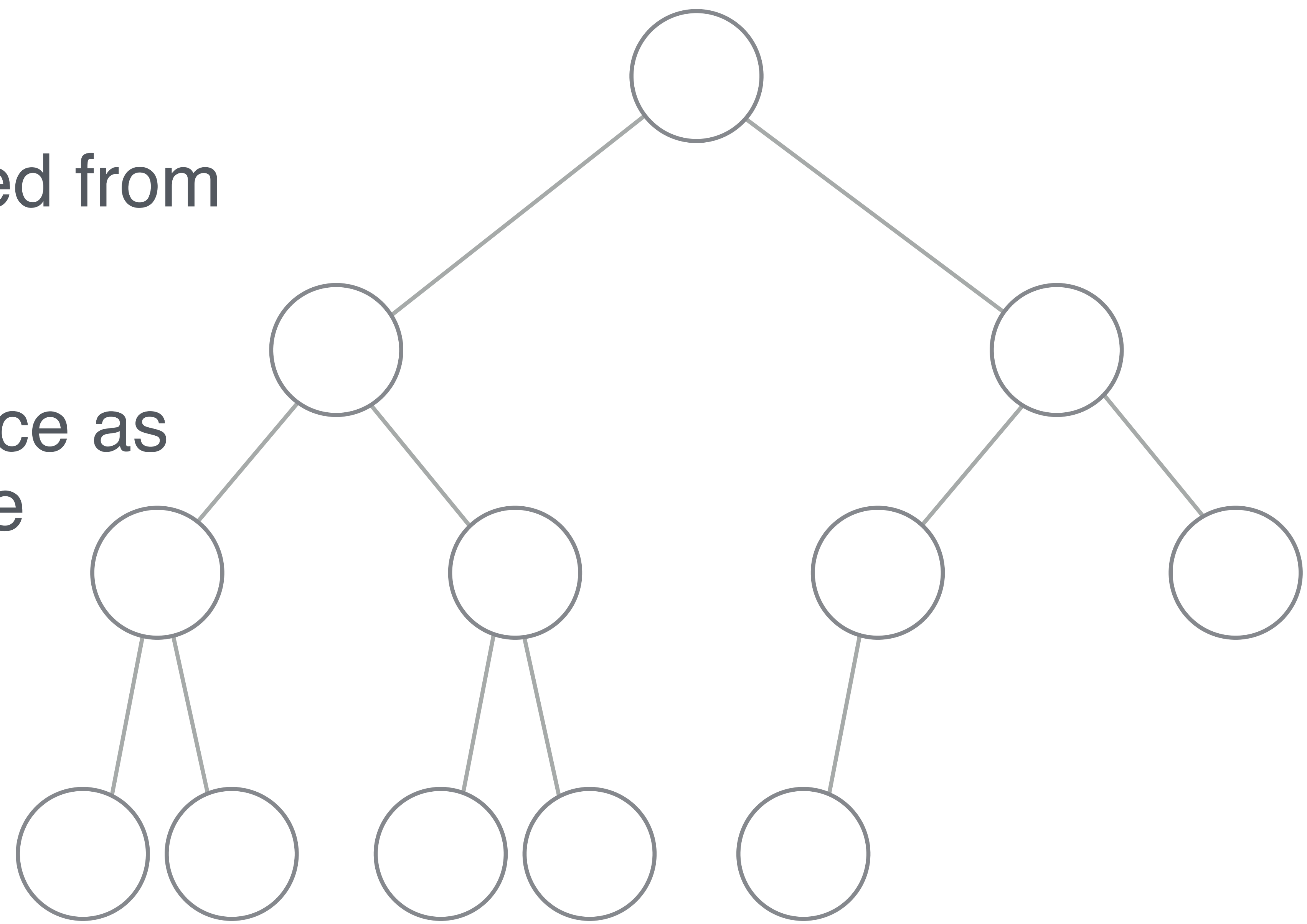
- option 3: a self-balancing BST
  - add: **O(logN)**
  - findMin: **O(logN)**
  - deleteMin: **O(logN)**

complete trees

-a **complete binary tree** has its levels completely filled, with the possible exception of the bottom level

-bottom level is filled from left to right

-each level has twice as many nodes as the previous level

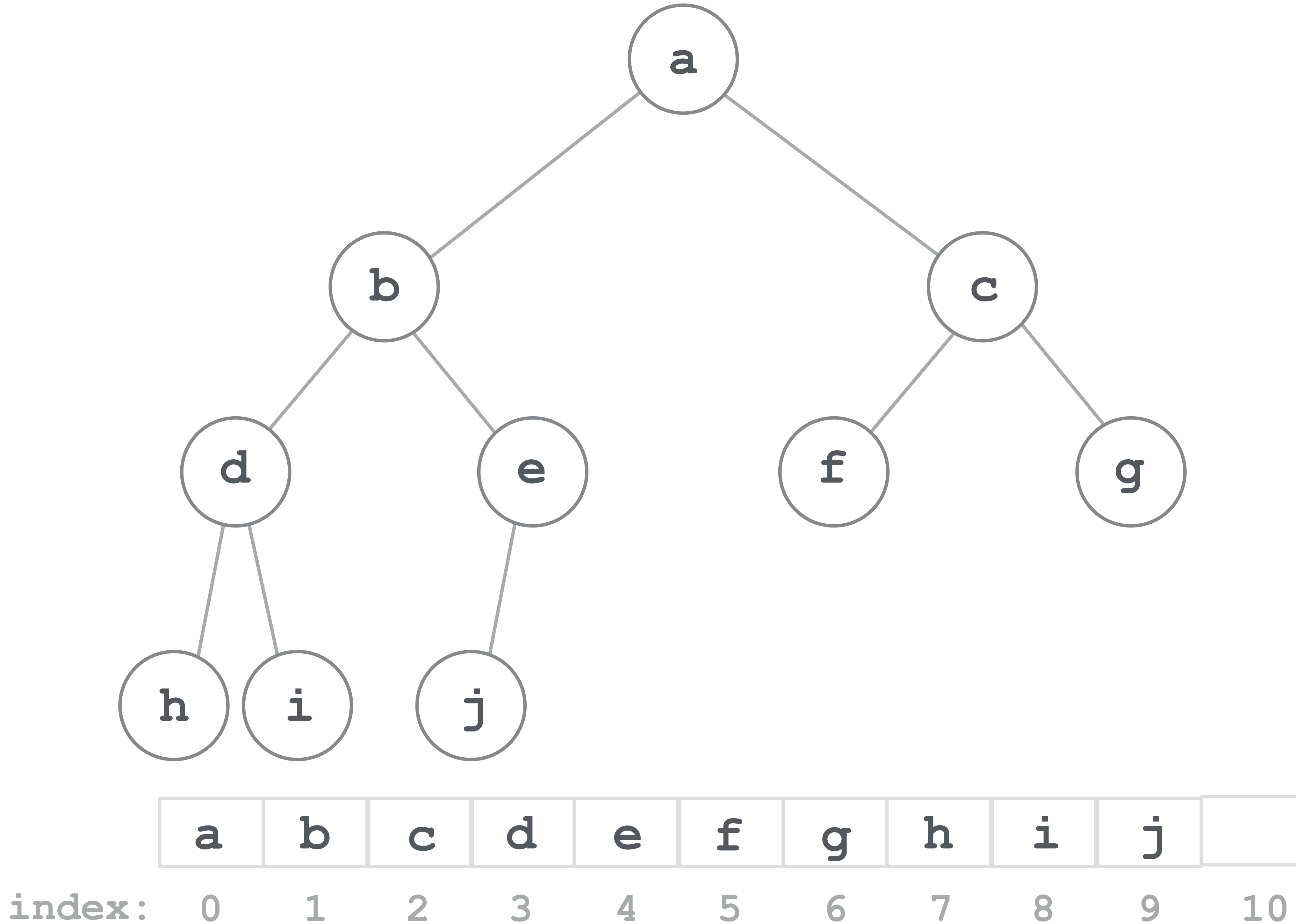


complete trees as an array

-if we are guaranteed that tree is complete, we can implement it as an array instead of a linked structure

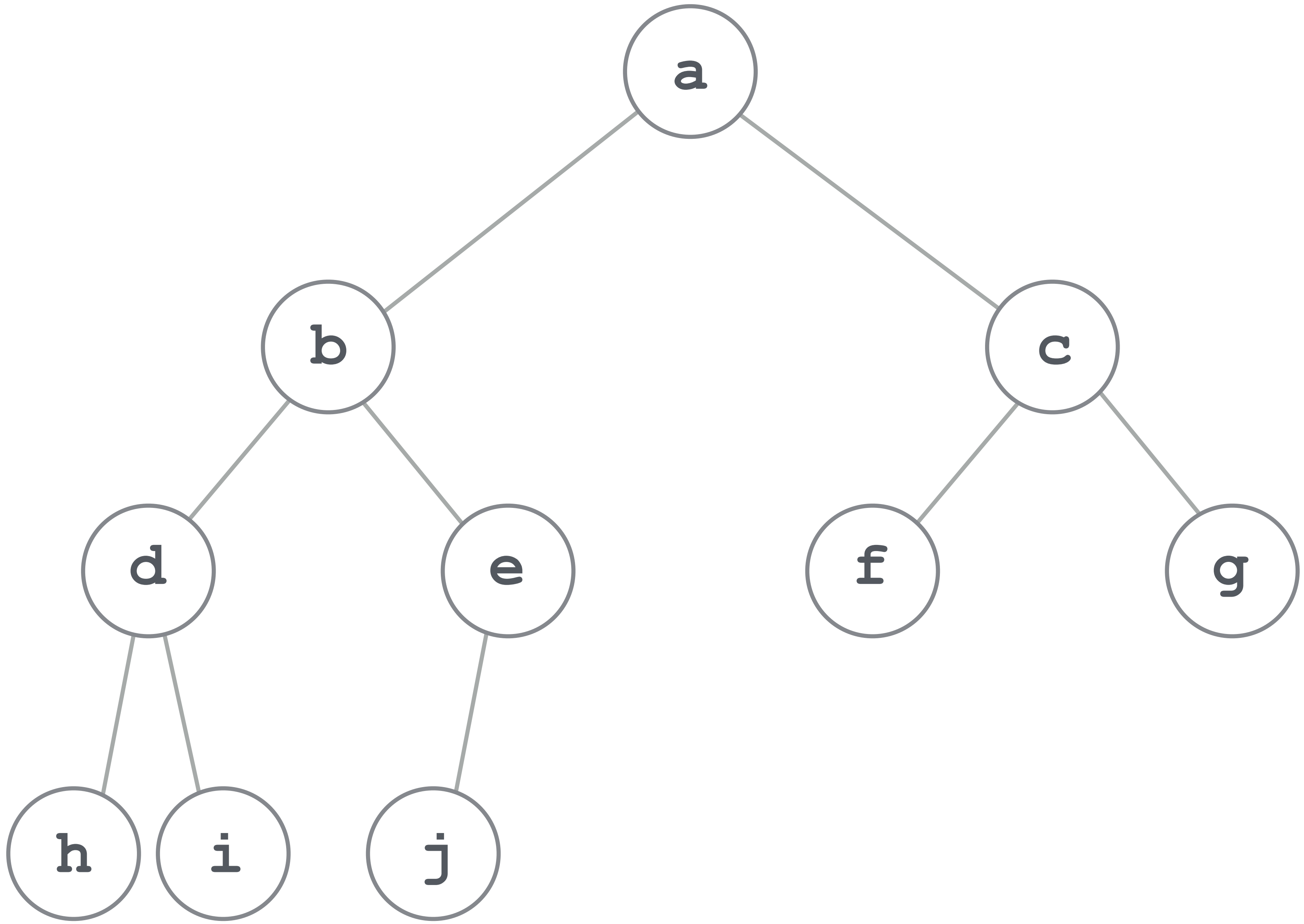
-the root goes at index 0, its left child at index 1, its right child at index 2

-for any node at index i, it two children are at index `(i*2) + 1` and `(i*2) + 2`



-for example, d’s children start at (3*2) + 1

-how can we compute the index of *any* node’s parent?



	a	b	c	d	e	f	g	h	i	j	
index:	0	1	2	3	4	5	6	7	8	9	10

-luckily, integer division automatically truncates

-any node’s parent is at index (i-1) / 2

complete trees as an array

-keep track of a `currentSize` variable

- -holds the total number of nodes in the tree
- -the very last leaf of the bottom level will be at index `currentSize - 1`

-when computing the index of a child node, if that index is `>= currentSize`, then the child does not exist

traversal helper methods

```
int leftChildIndex(int i) {
  return (i*2) + 1;
}


int rightChildIndex(int i) {
  return (i*2) + 2;
}


int parentIndex(int i) {
  return (i-1) / 2;
}
```

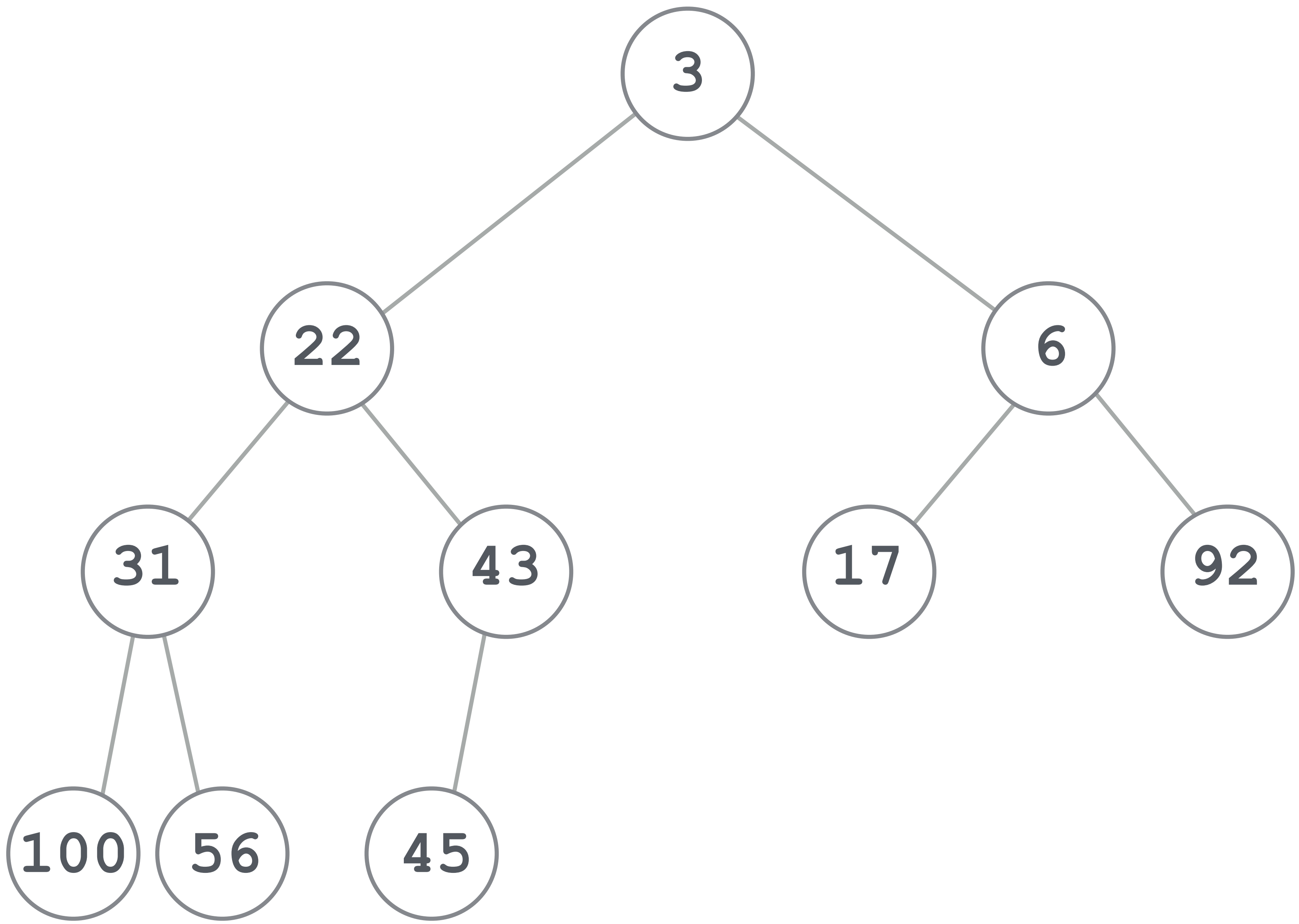
binary heap

-a **binary heap** is a binary tree with two special properties

- *structure:* it is a complete tree
- *order:* the data in any node is less than or equal to the data of its children

-this is also called a **min-heap**

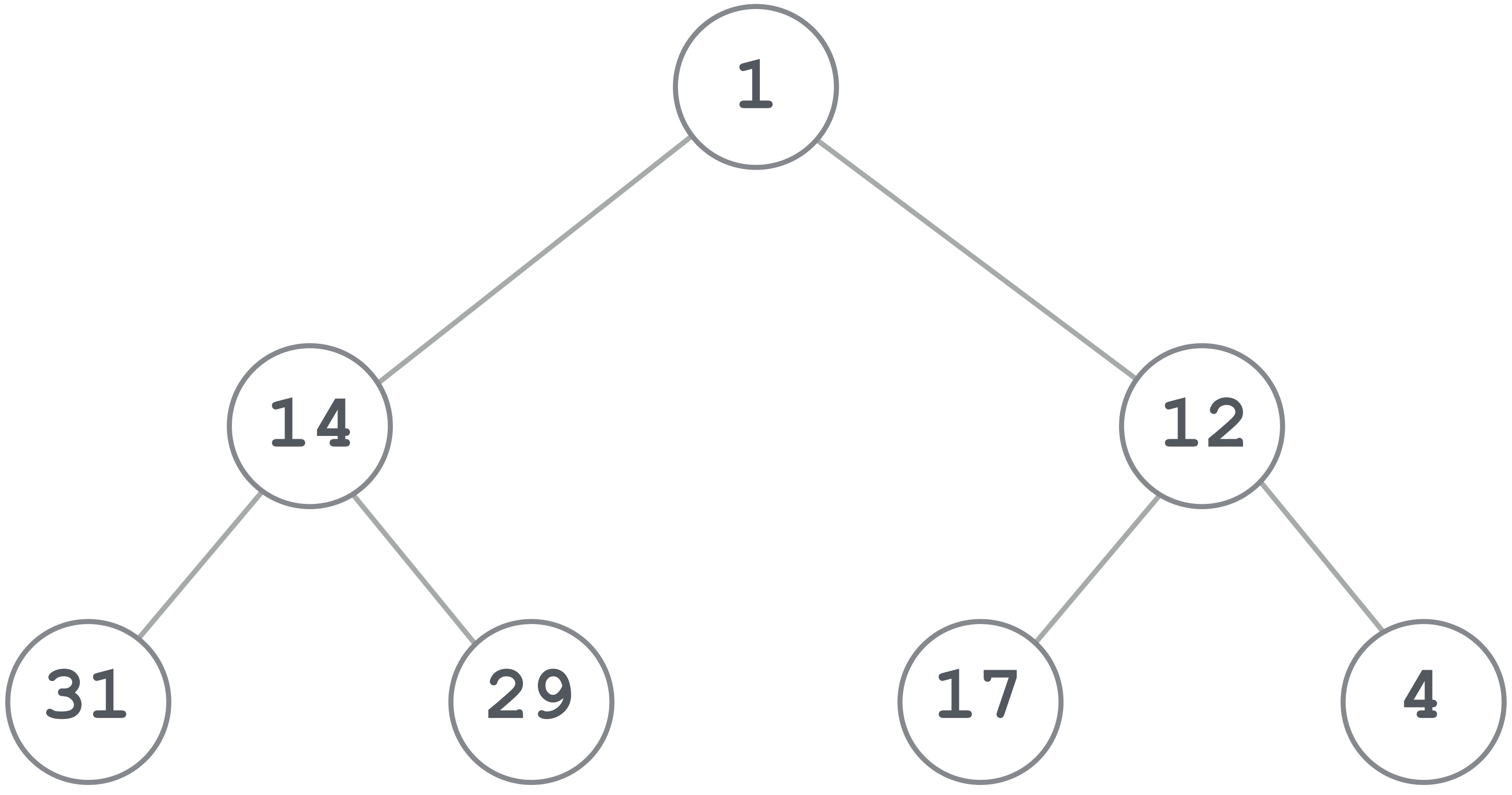
-a **max-heap** would have the opposite property



-order of children does not matter, only that they are greater than their parent

IS THIS A MIN-HEAP?

A) **yes**
B) **no**



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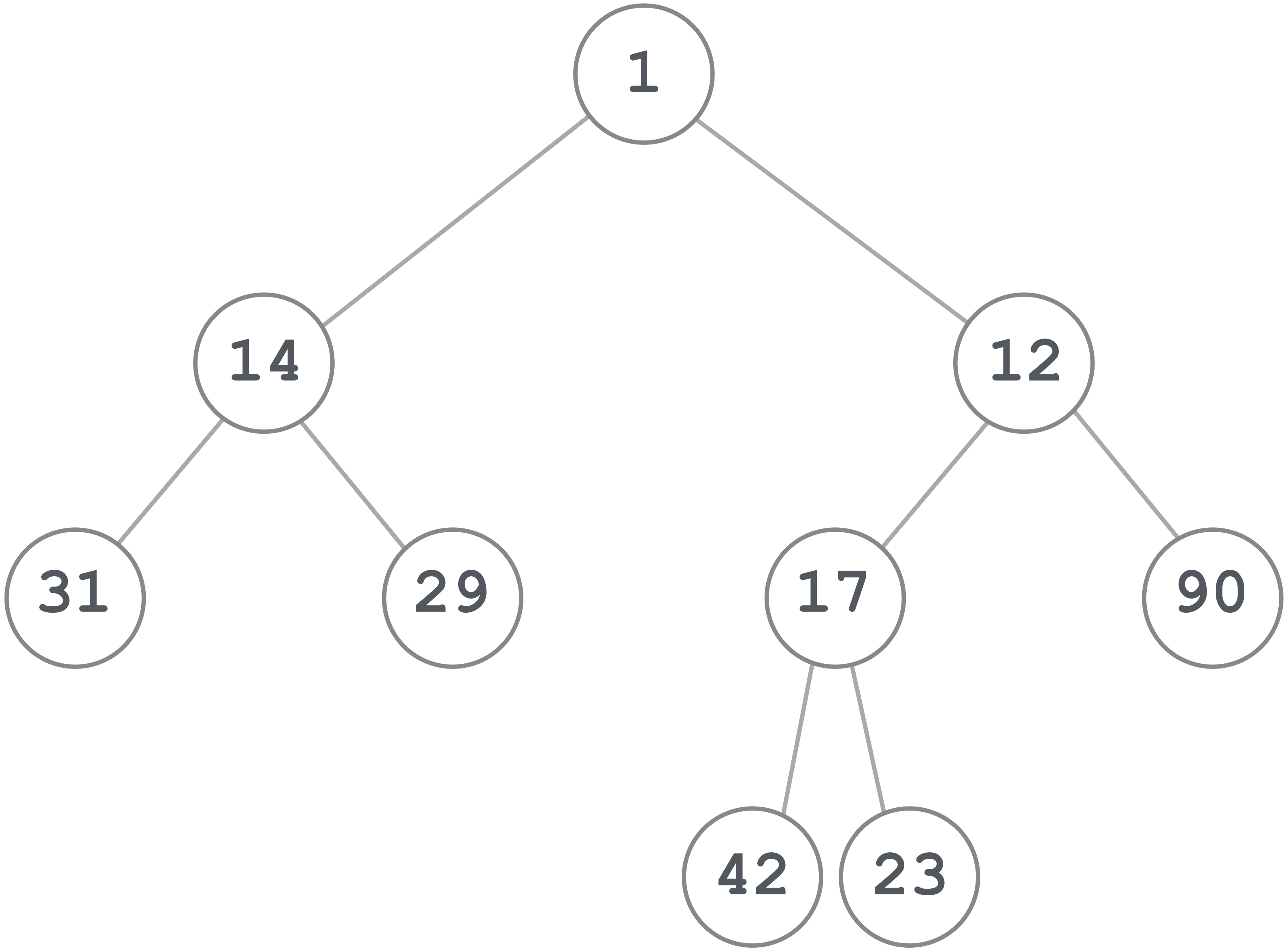


adding to a heap

-we must be careful to maintain the two properties when adding to a heap

- -structure and order

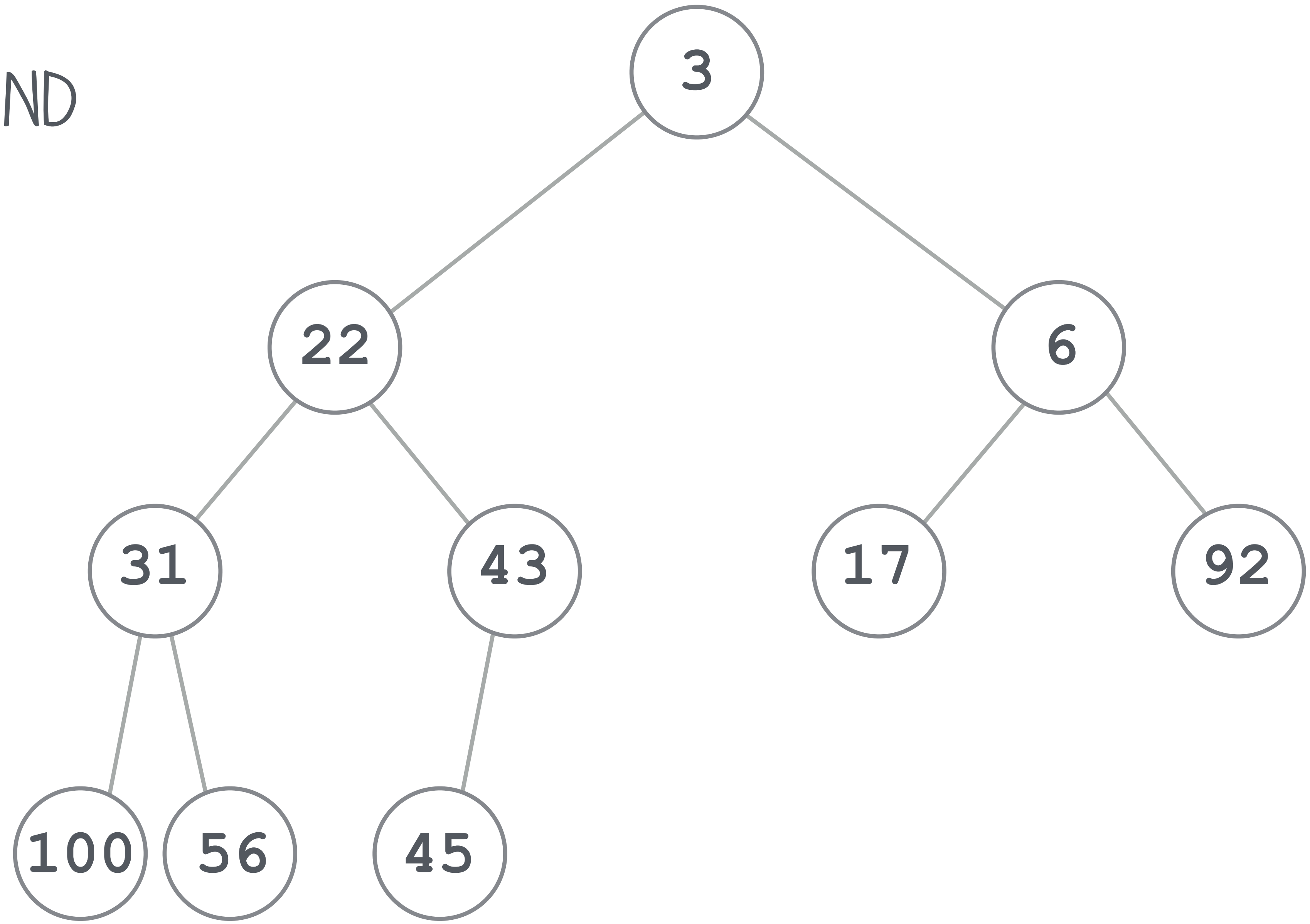
-deal with the structure property first… where can the new item go to maintain a complete tree?

-then, *percolate* the item upward until the order property is restored

- -swap upwards until > parent

ADDING 14

PUT IT AT THE END OF THE TREE



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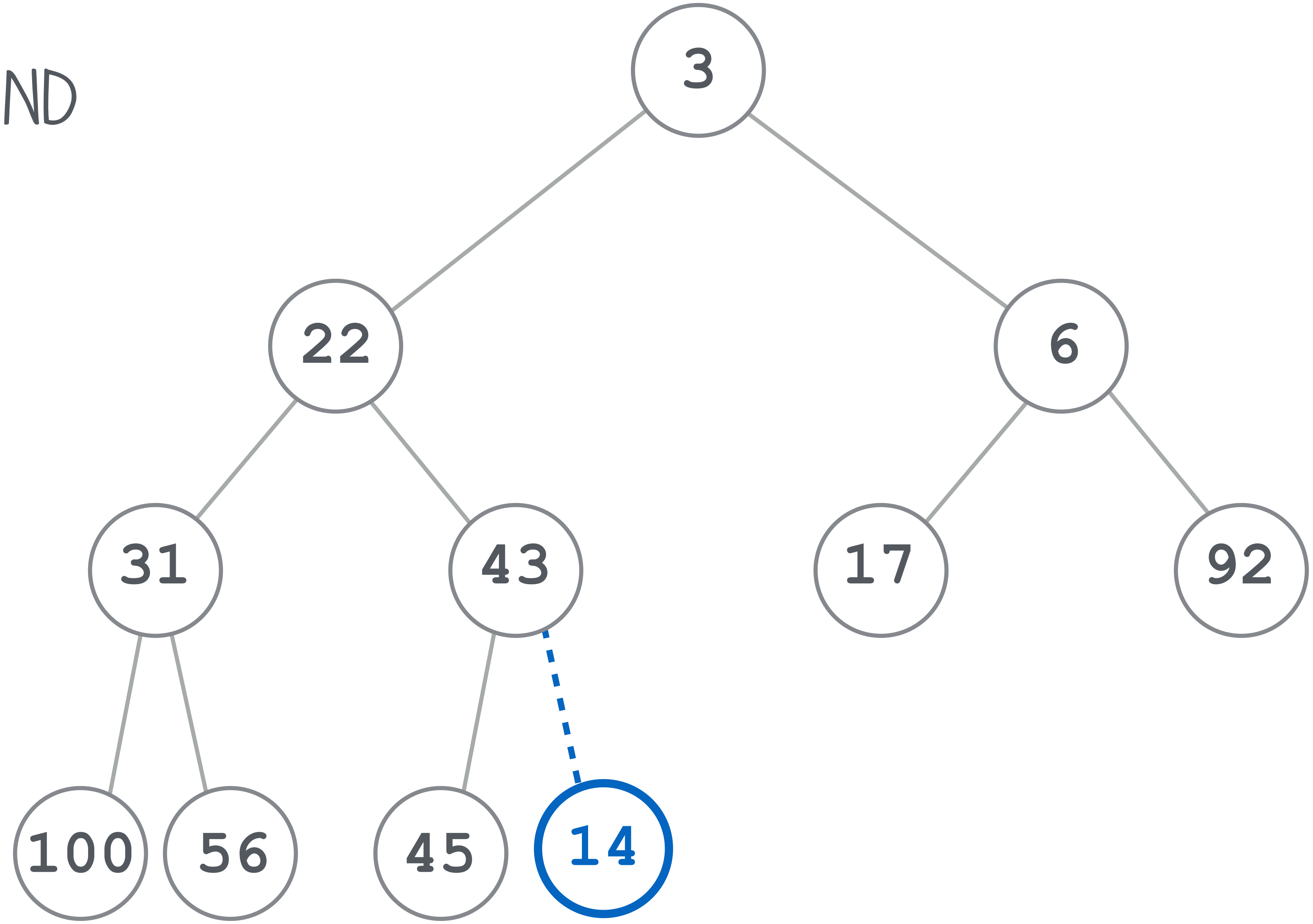
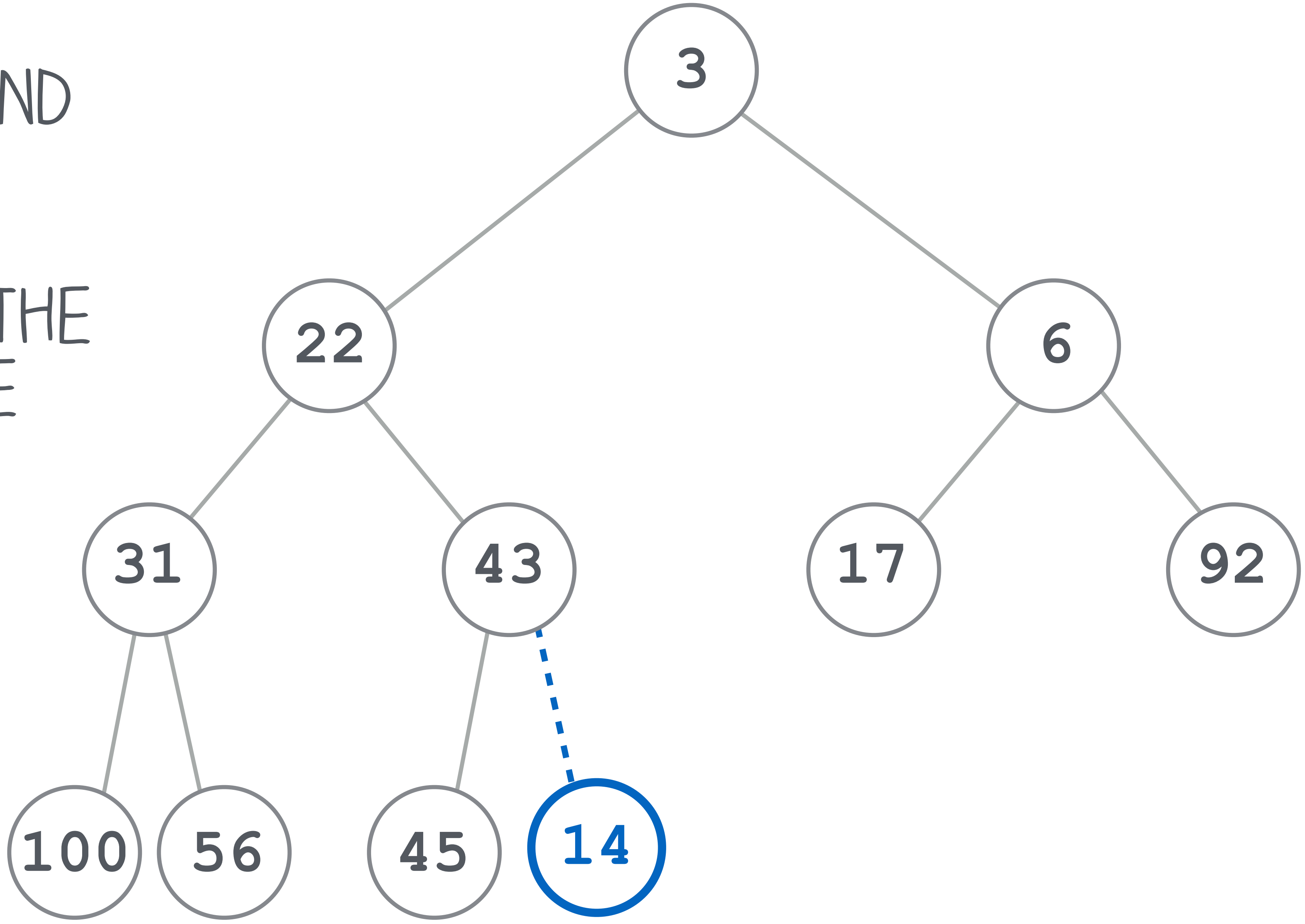


ADDING 14

PUT IT AT THE END OF THE TREE

PERCOLATE UP THE TREE TO FIX THE ORDER



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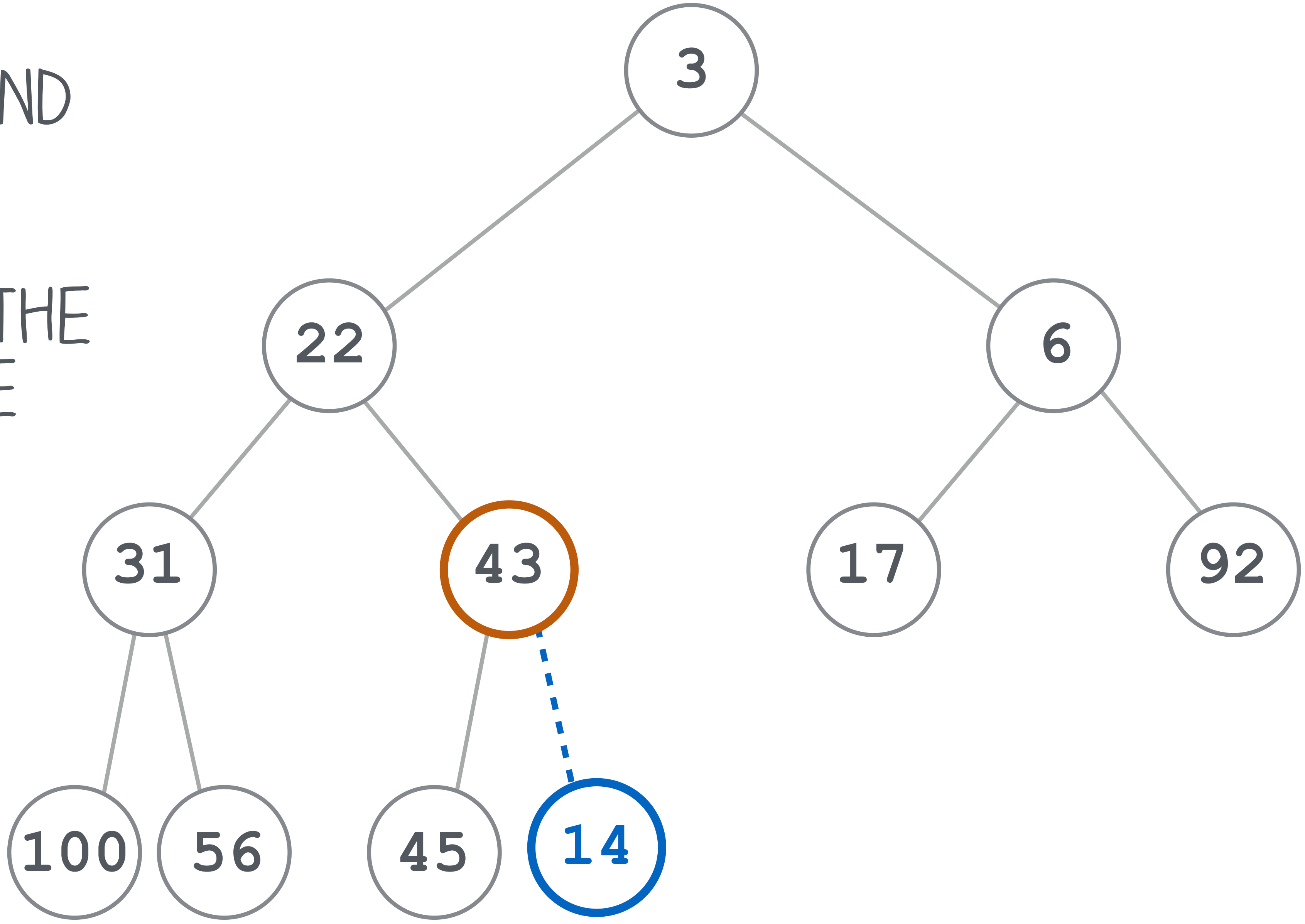

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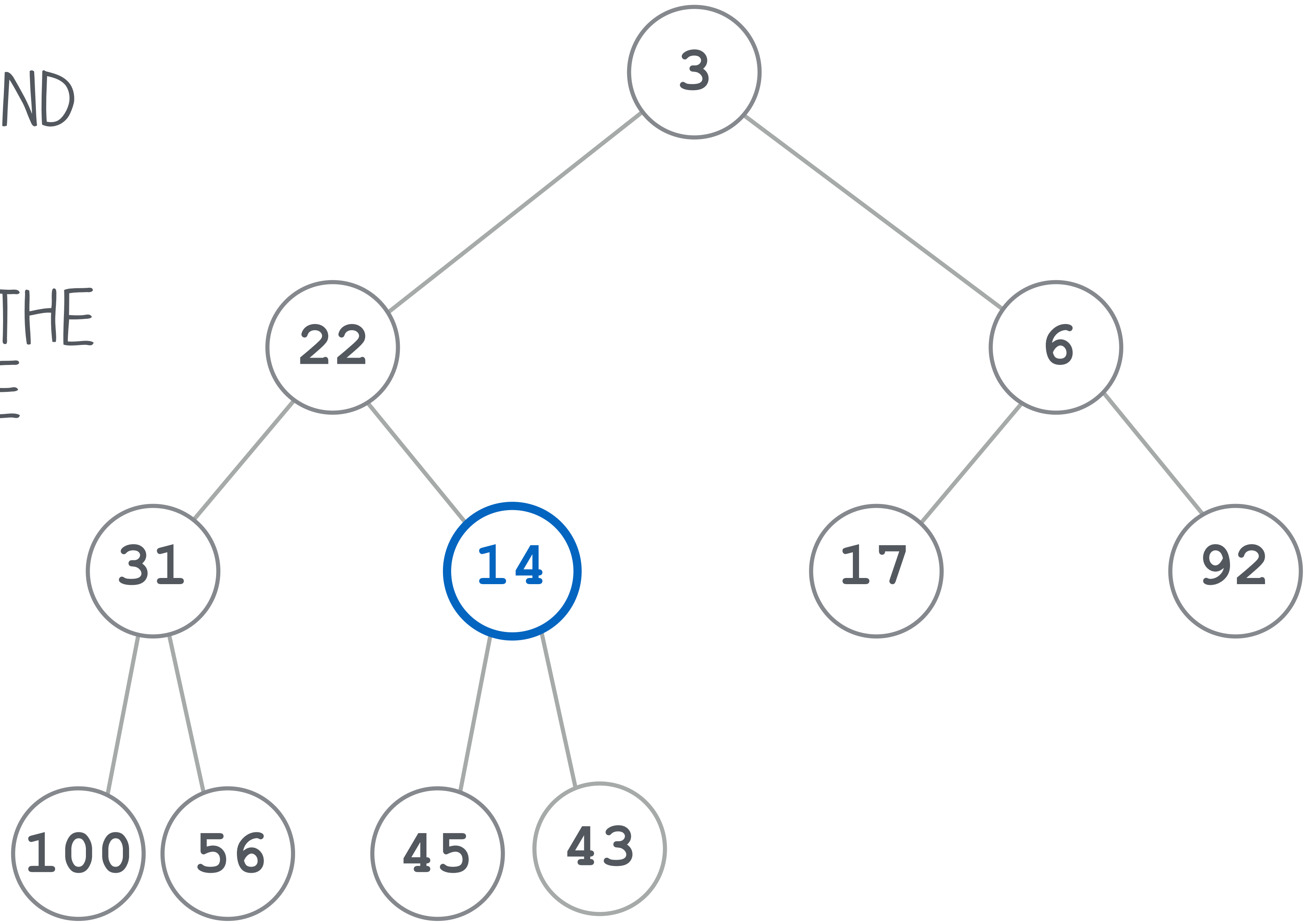

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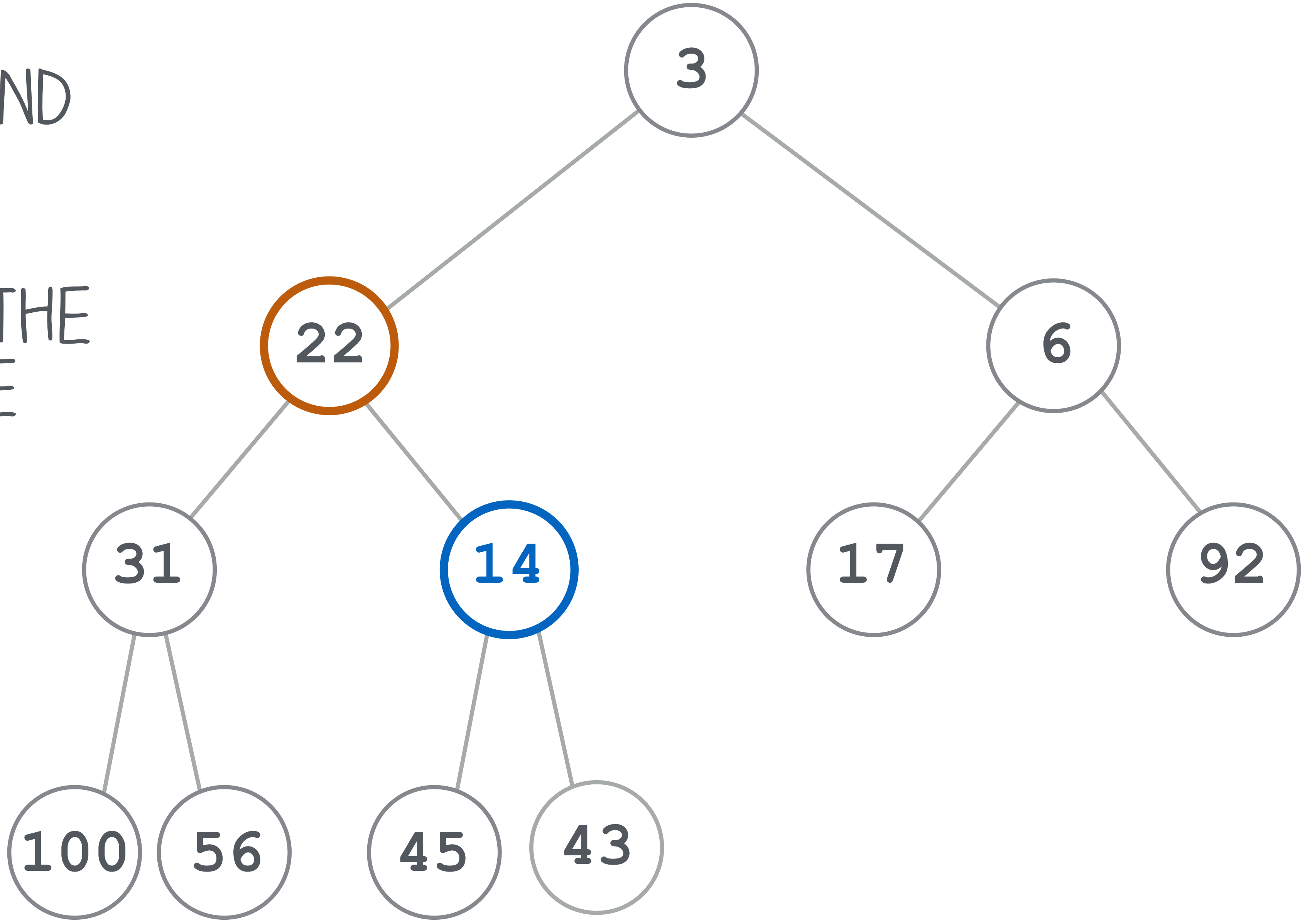

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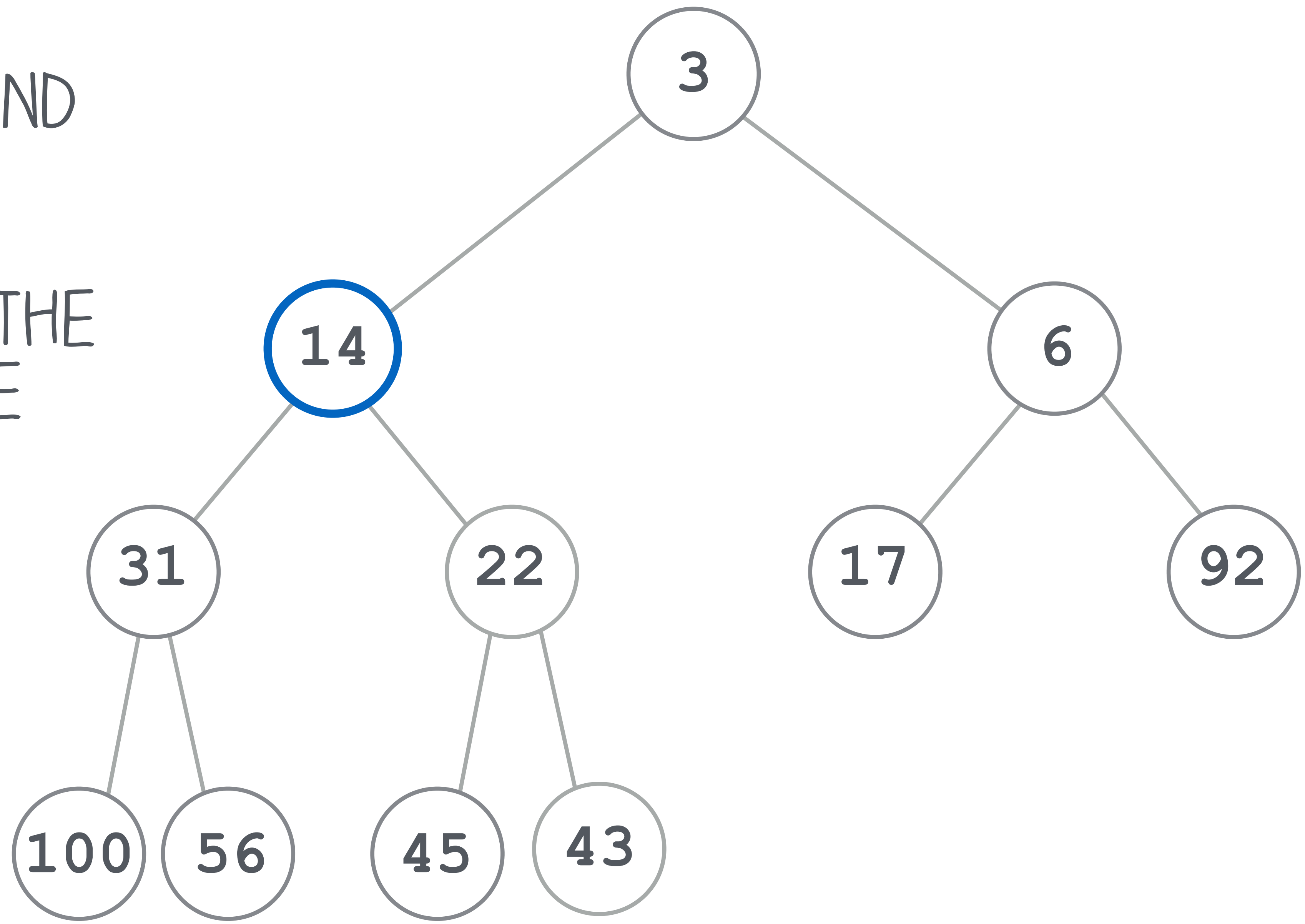

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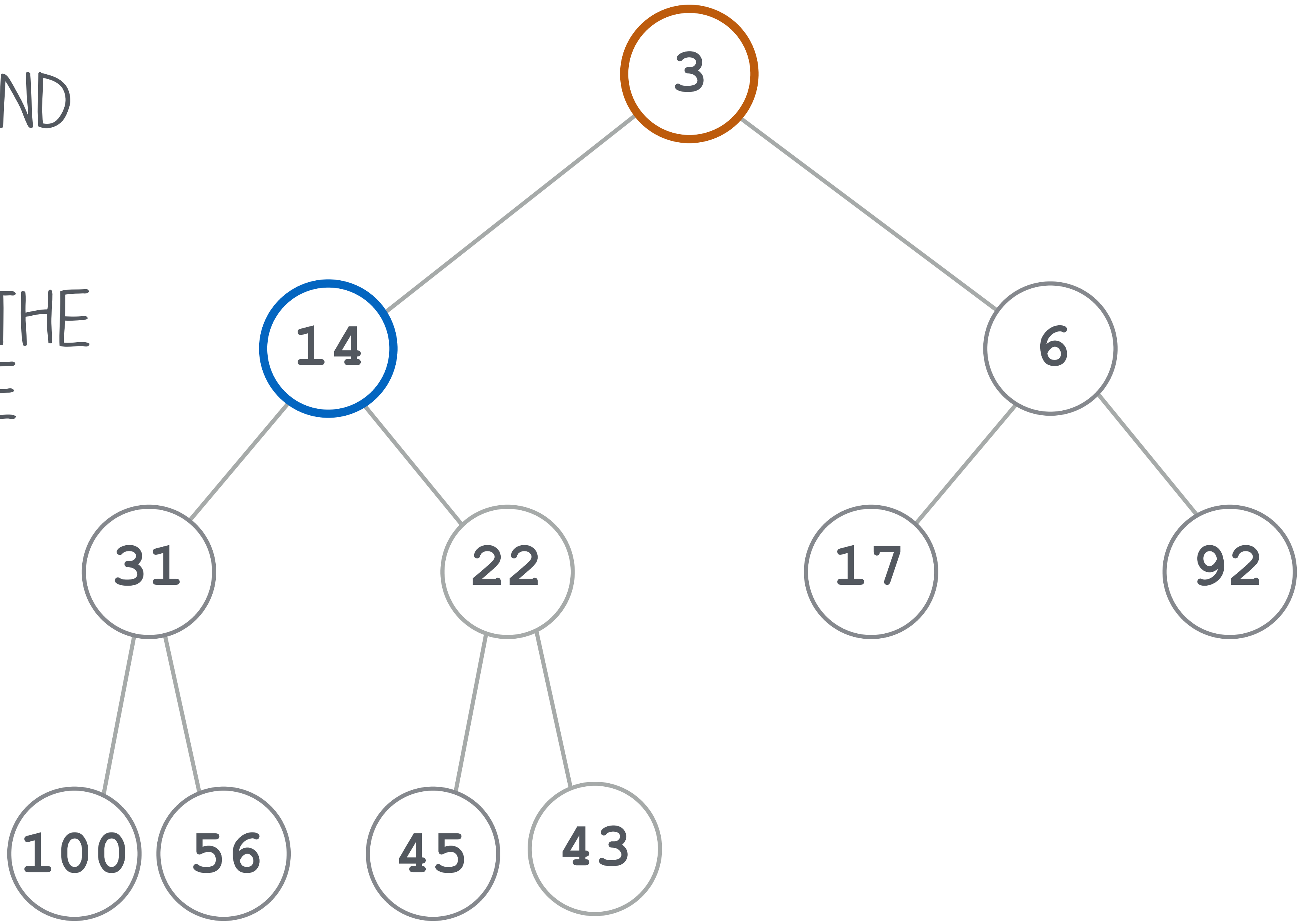

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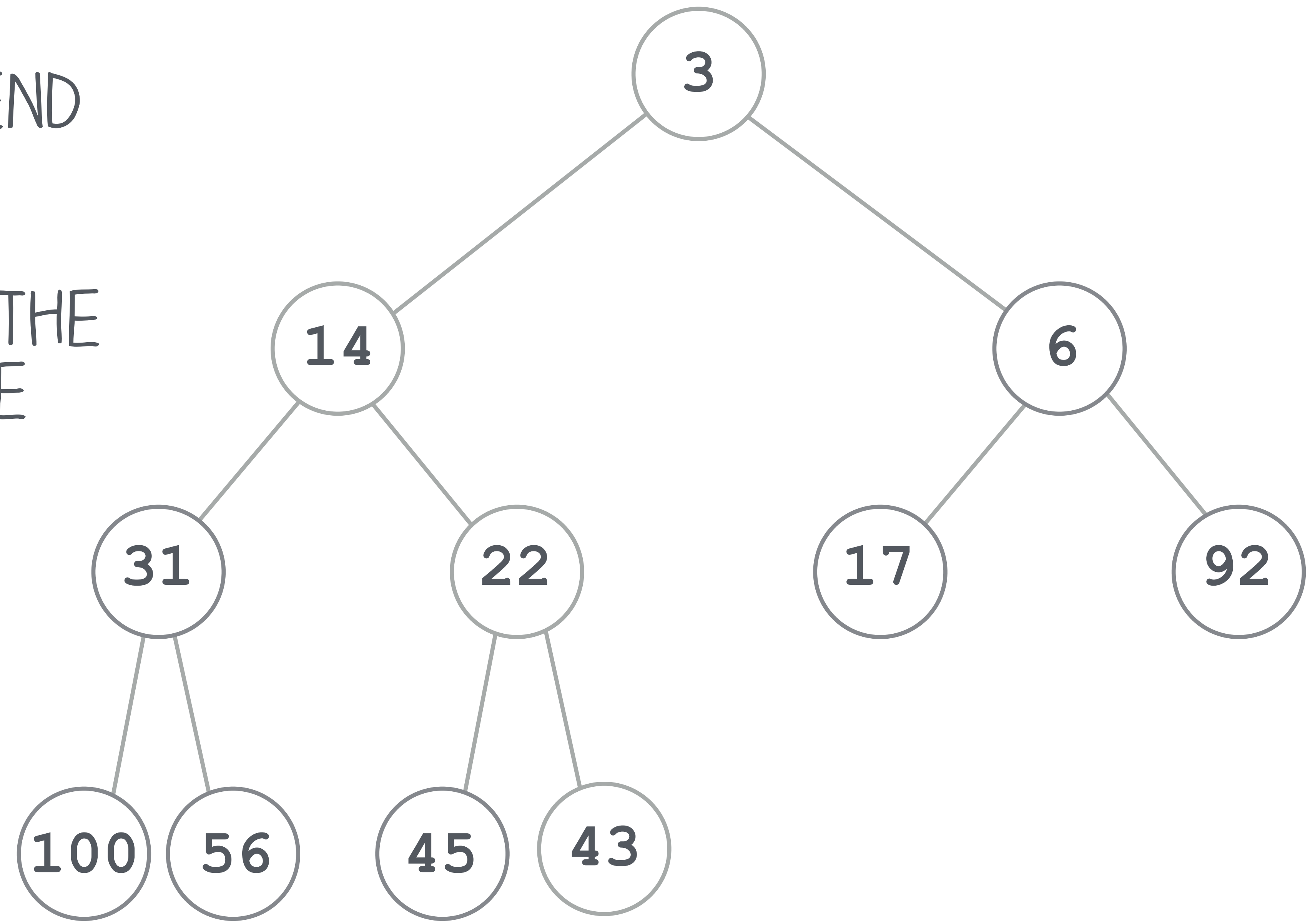
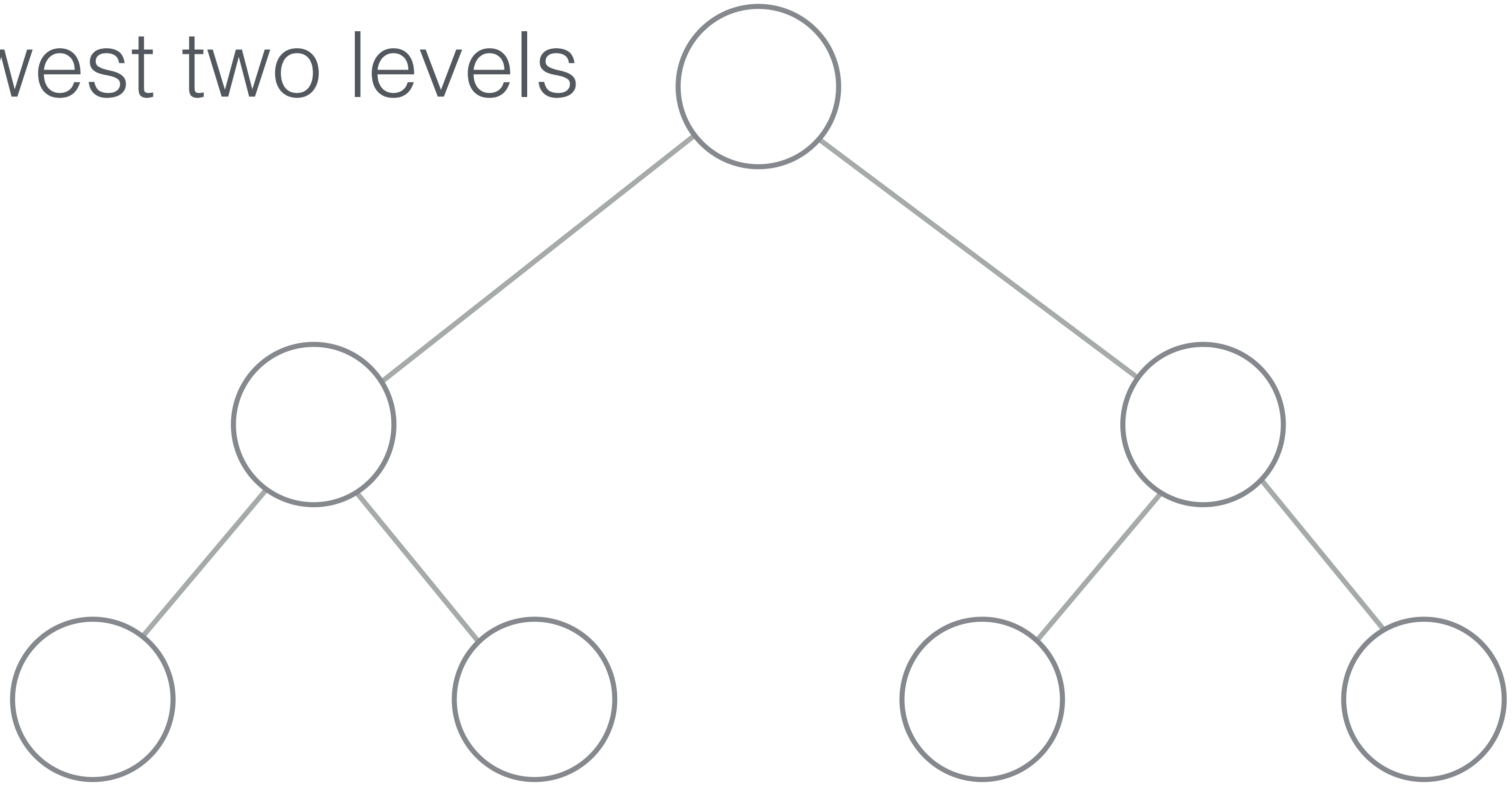


cost of add

-percolate up until smaller than all nodes below it…

-how many nodes are there on each level (in terms of N)?

- -about half on the lowest level
- -about 3/4 in the lowest two levels



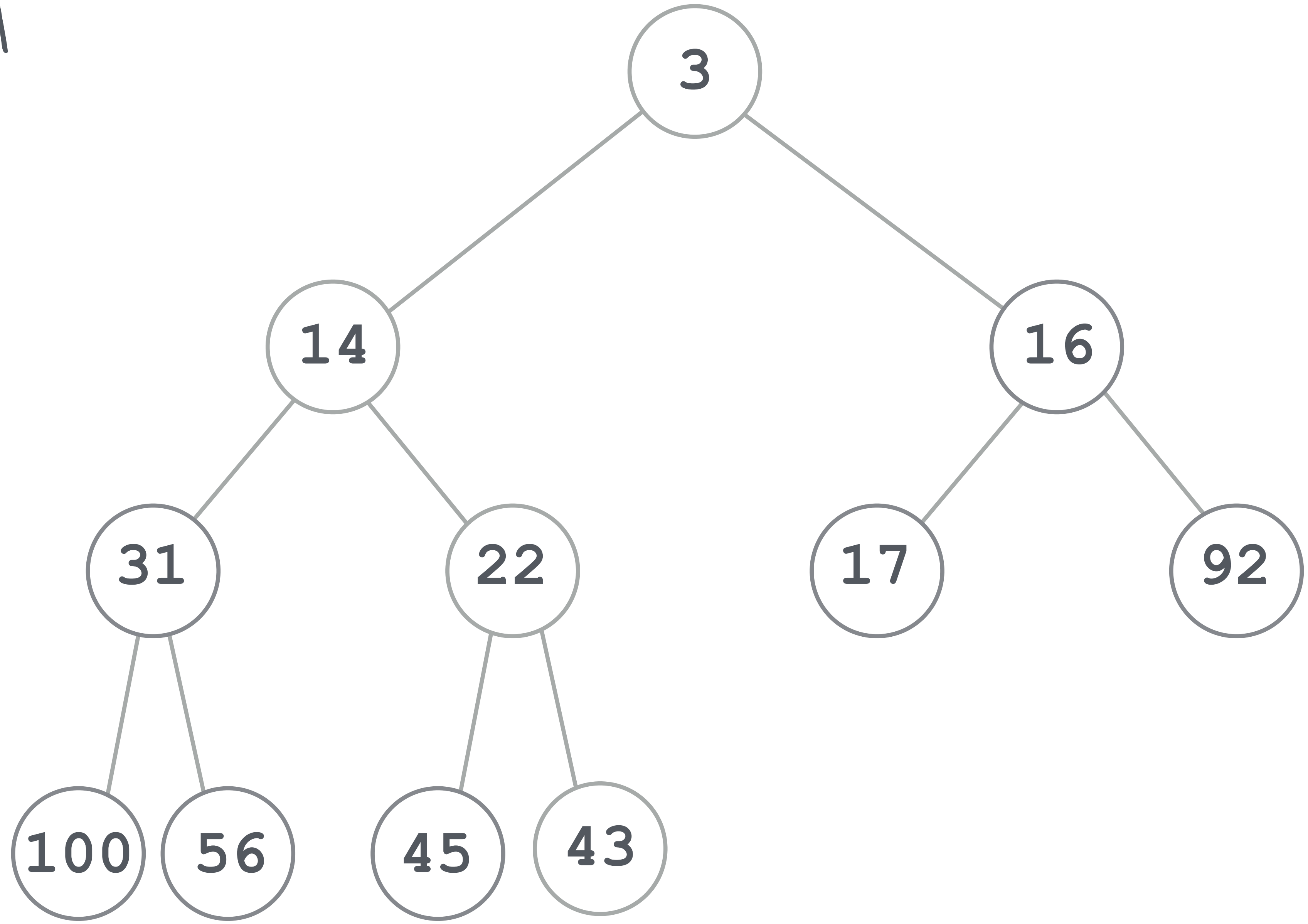
- if the new item is the smallest in the set, cost is **O(logN)**
  - must percolate up every level to the root
  - complete trees have logN levels
    - *is this the worst, average, or best case?*

- it has been shown that on average, 2.6 comparisons are needed for any N
  - thus, add terminates early, and average cost is **O(1)**

remove

LET'S REMOVE THE SMALLEST ITEM

TAKE OUT 3



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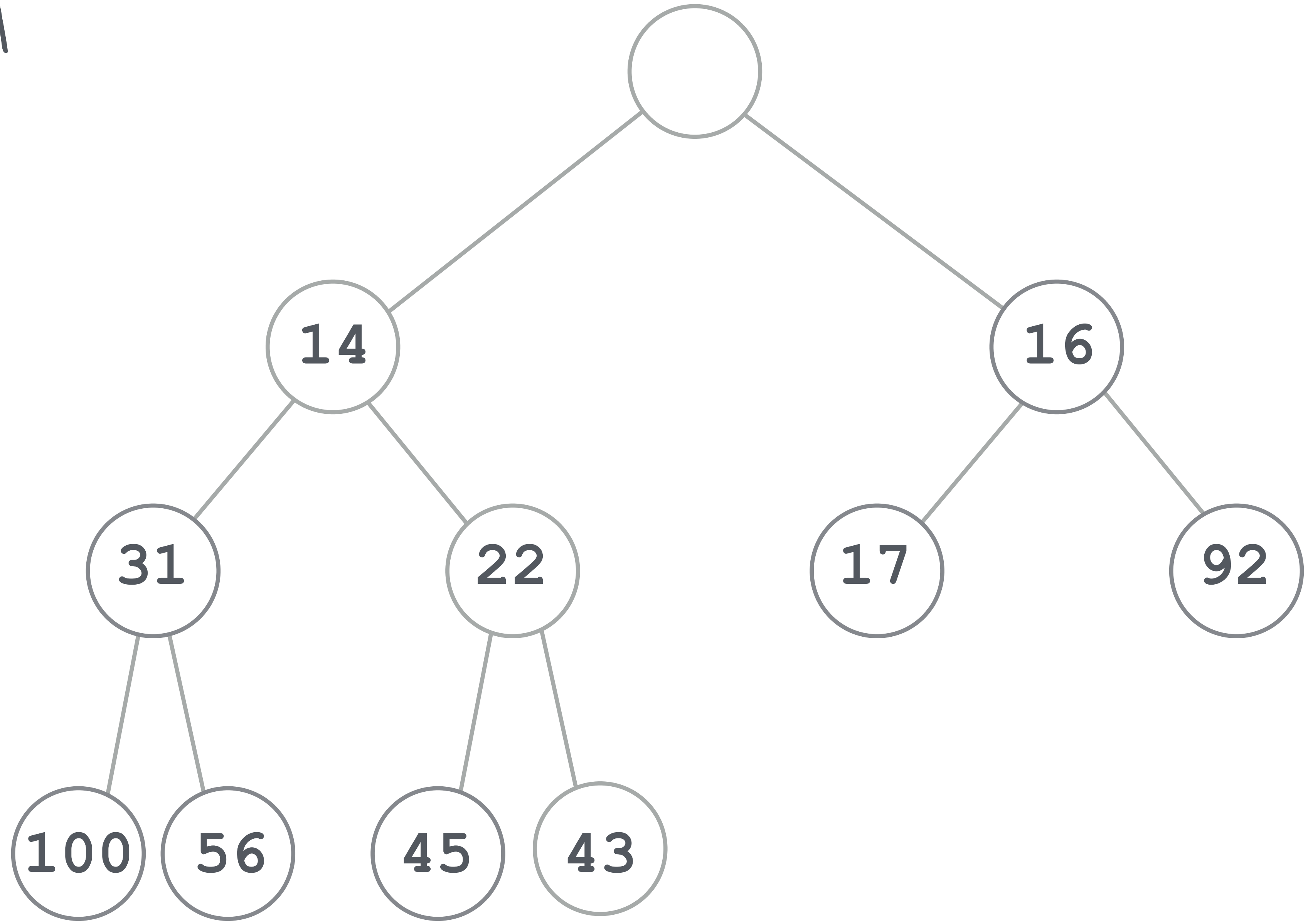
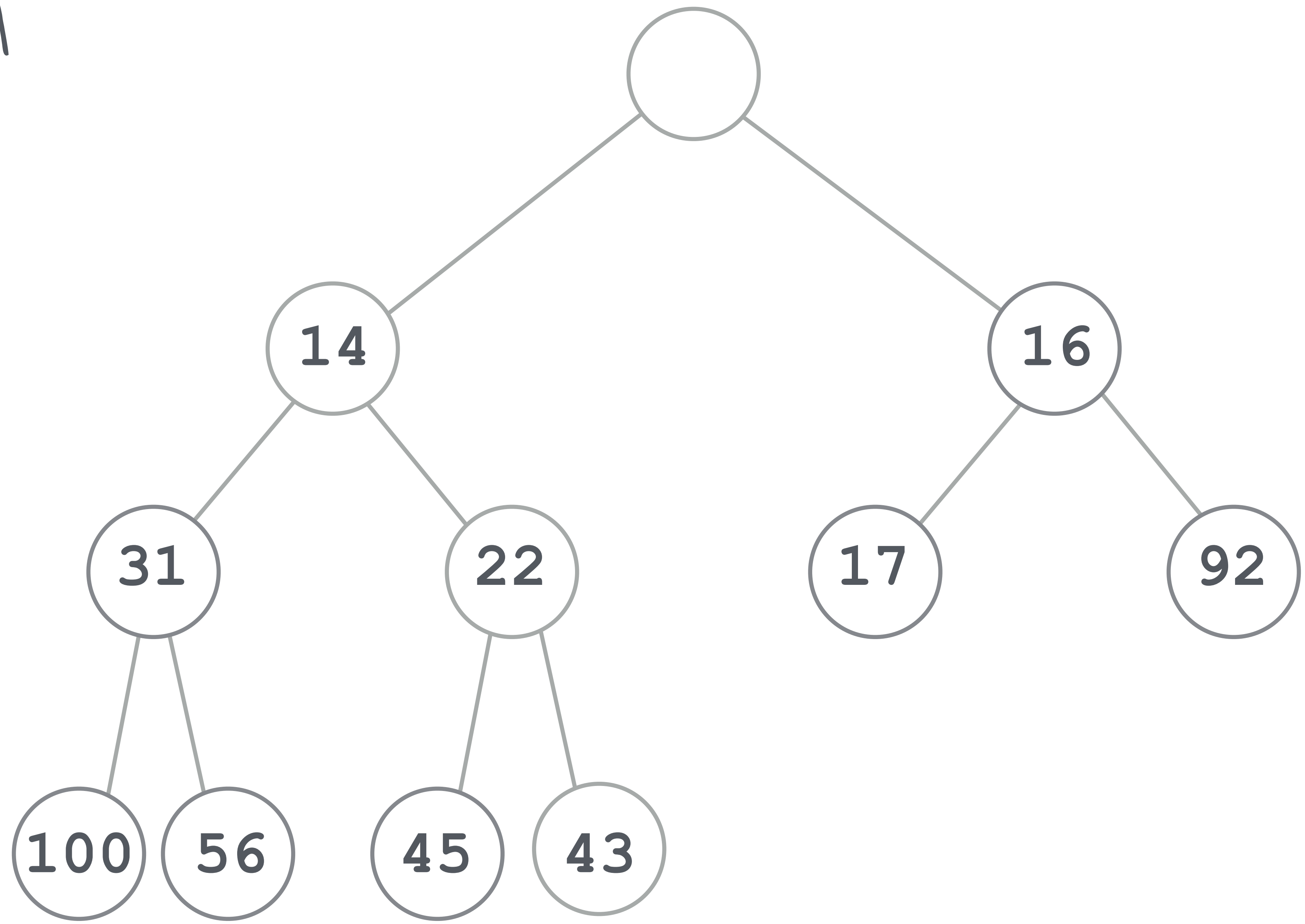


LET'S REMOVE THE SMALLEST ITEM

TAKE OUT 3

FILL WITH LAST ITEM ON LAST LEVEL. WHY?



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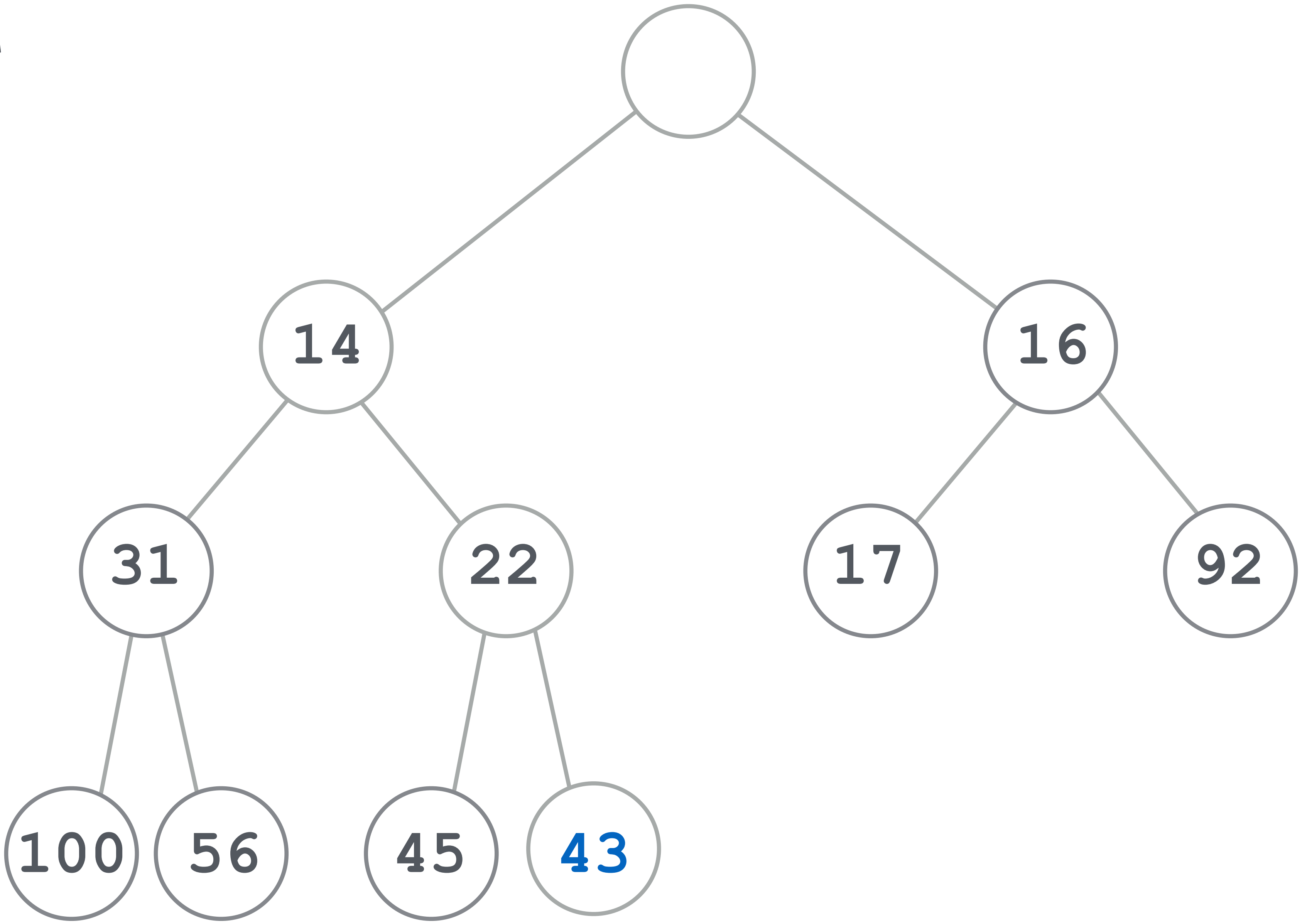

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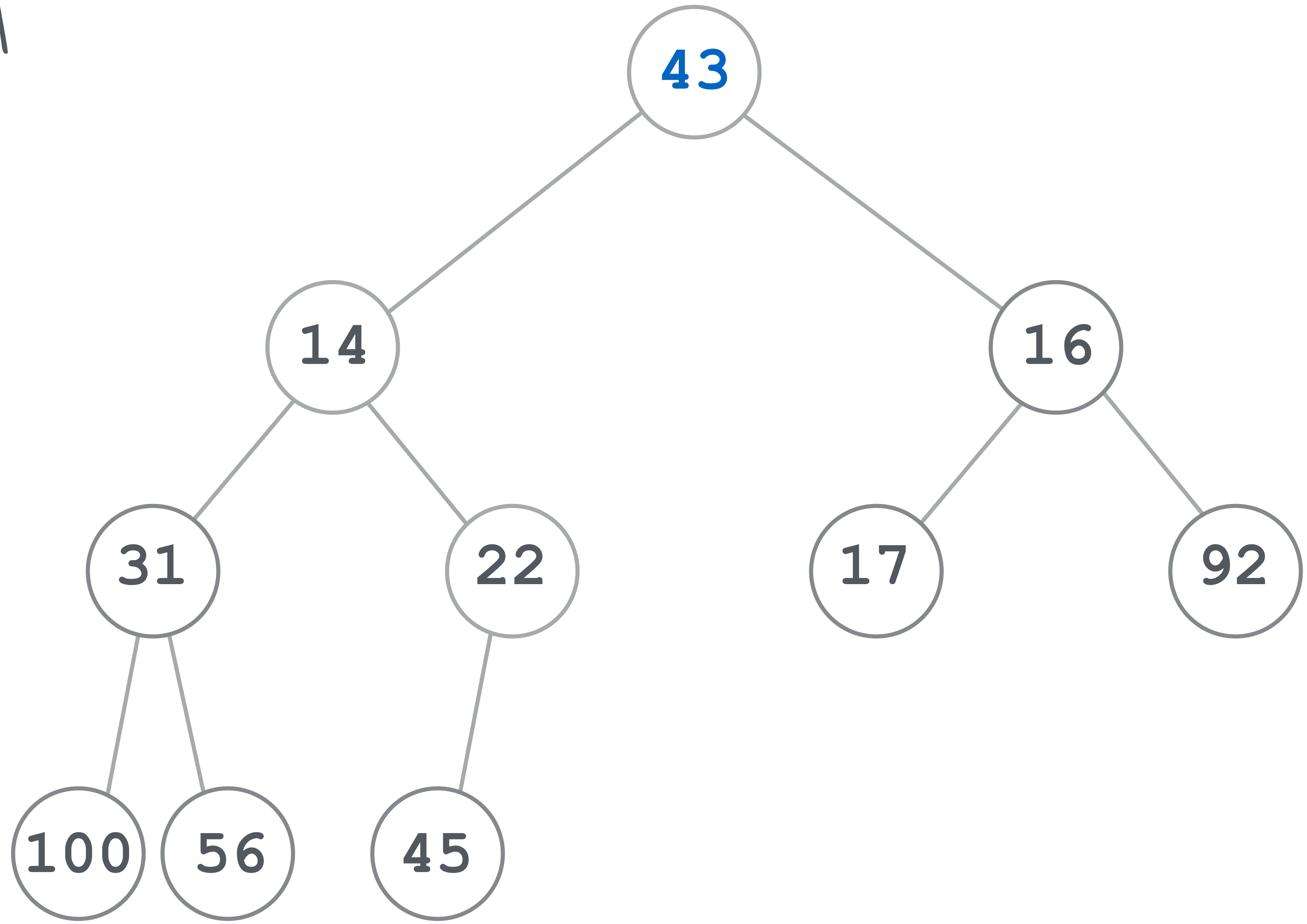
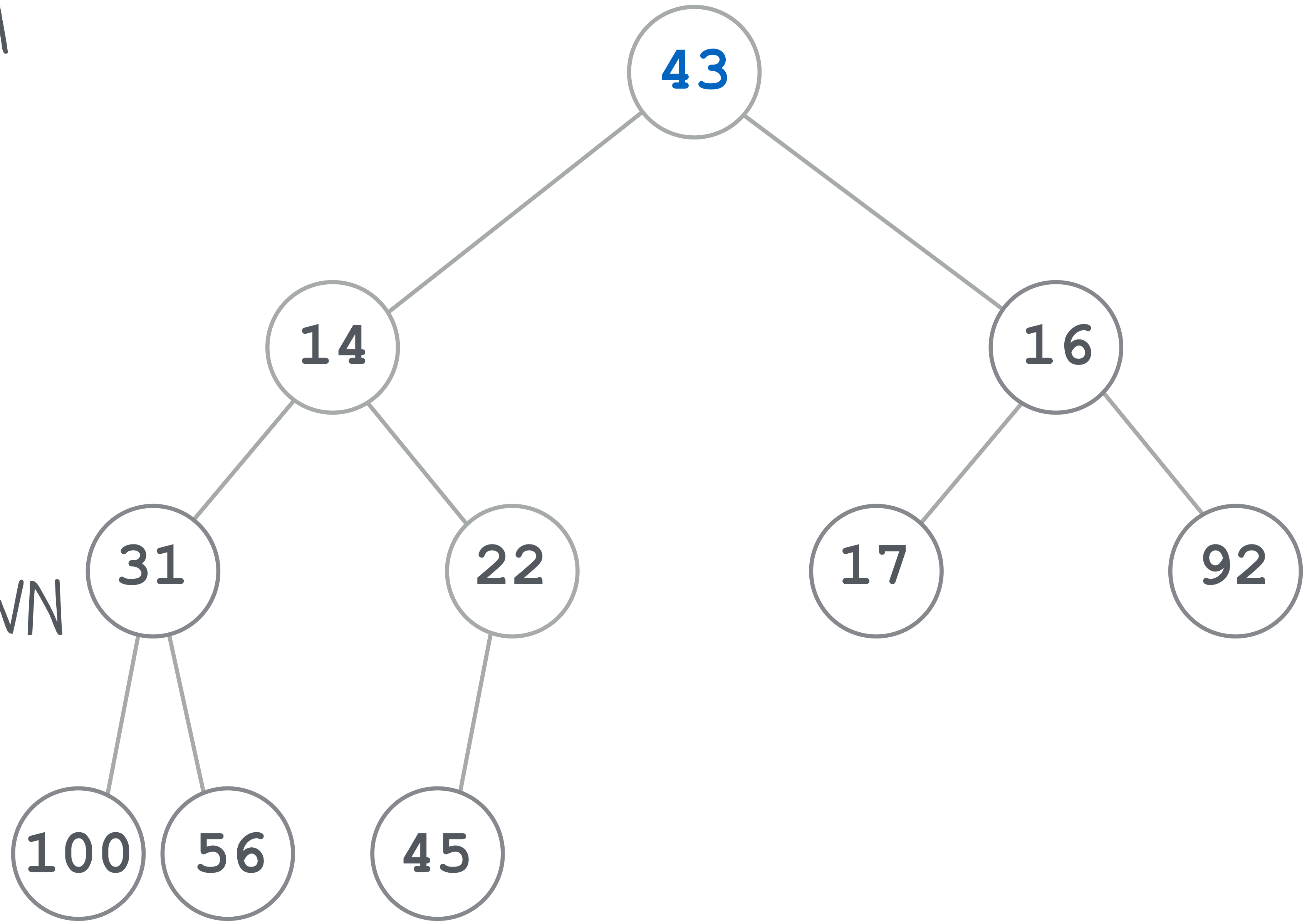


LET'S REMOVE THE SMALLEST ITEM

TAKE OUT 3

FILL WITH LAST ITEM ON LAST LEVEL. WHY?

PERCOLATE DOWN



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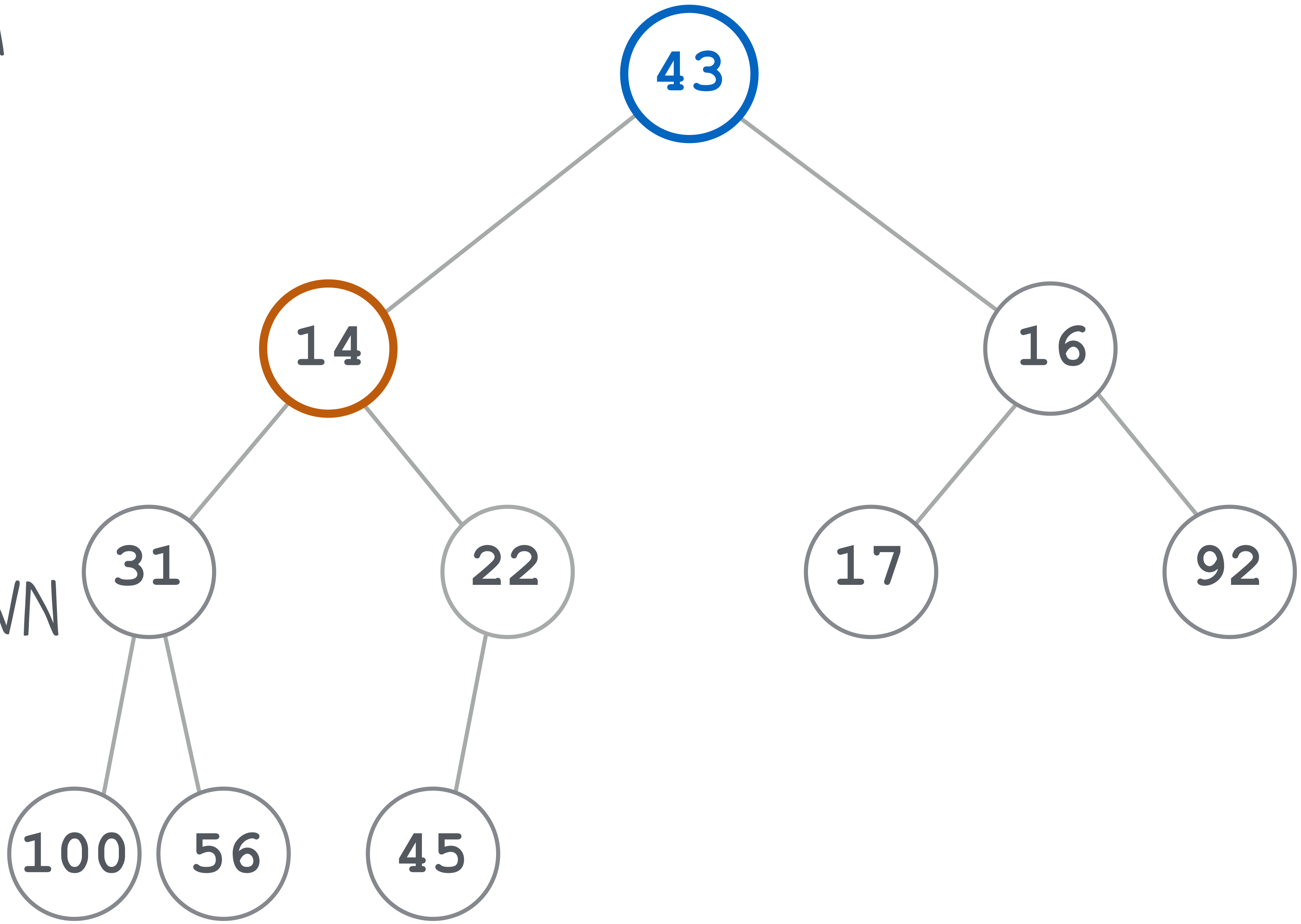

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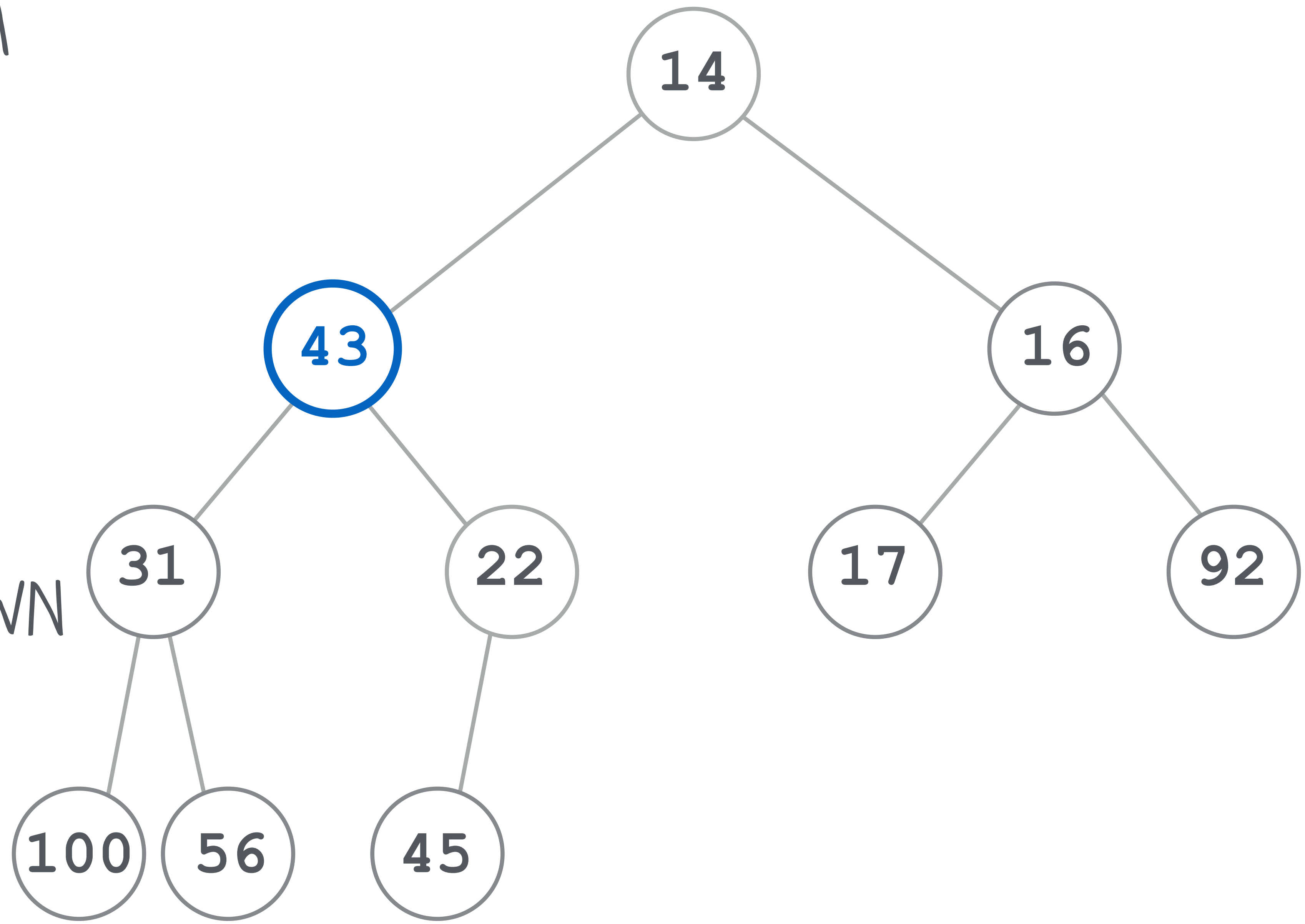

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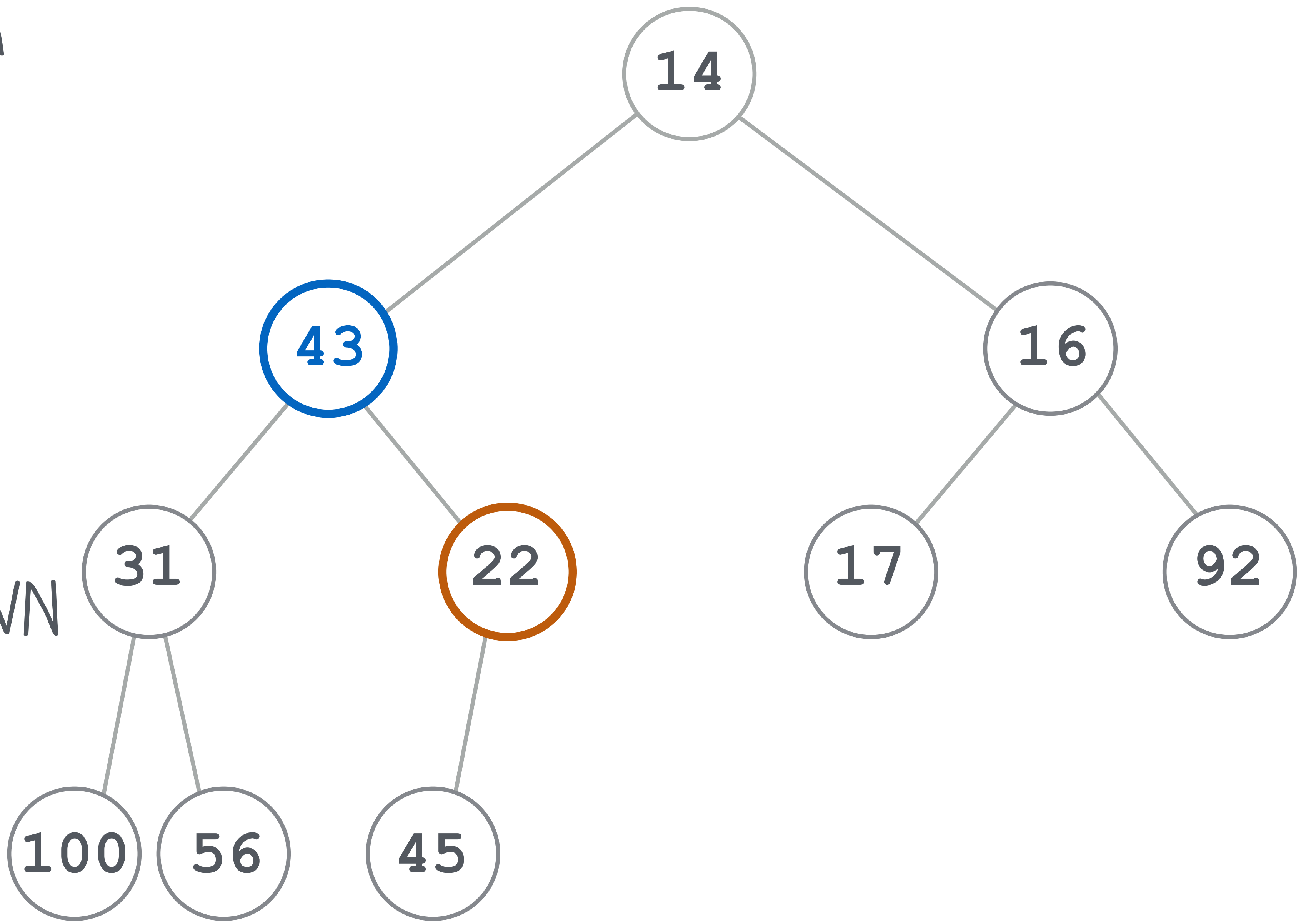

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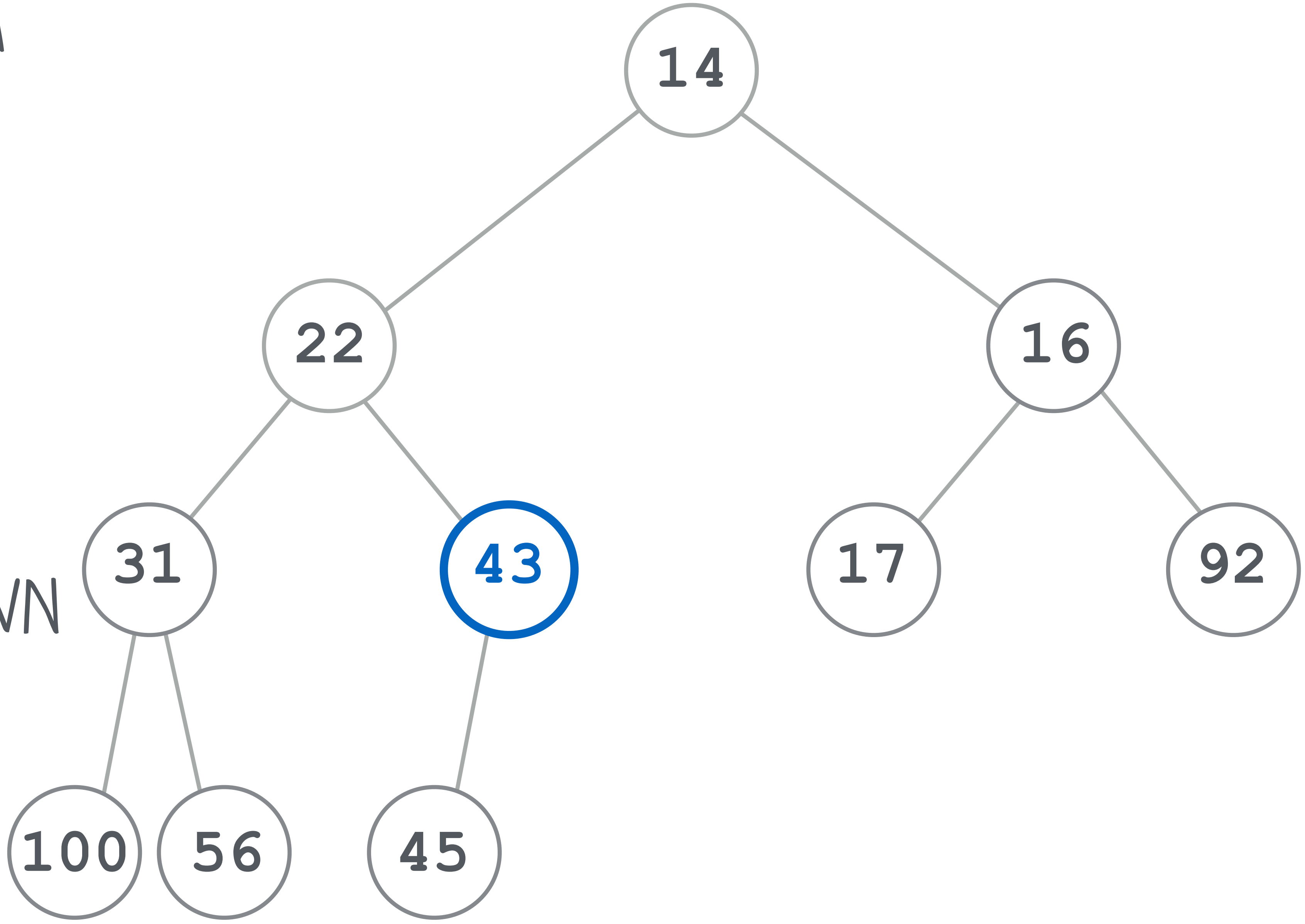

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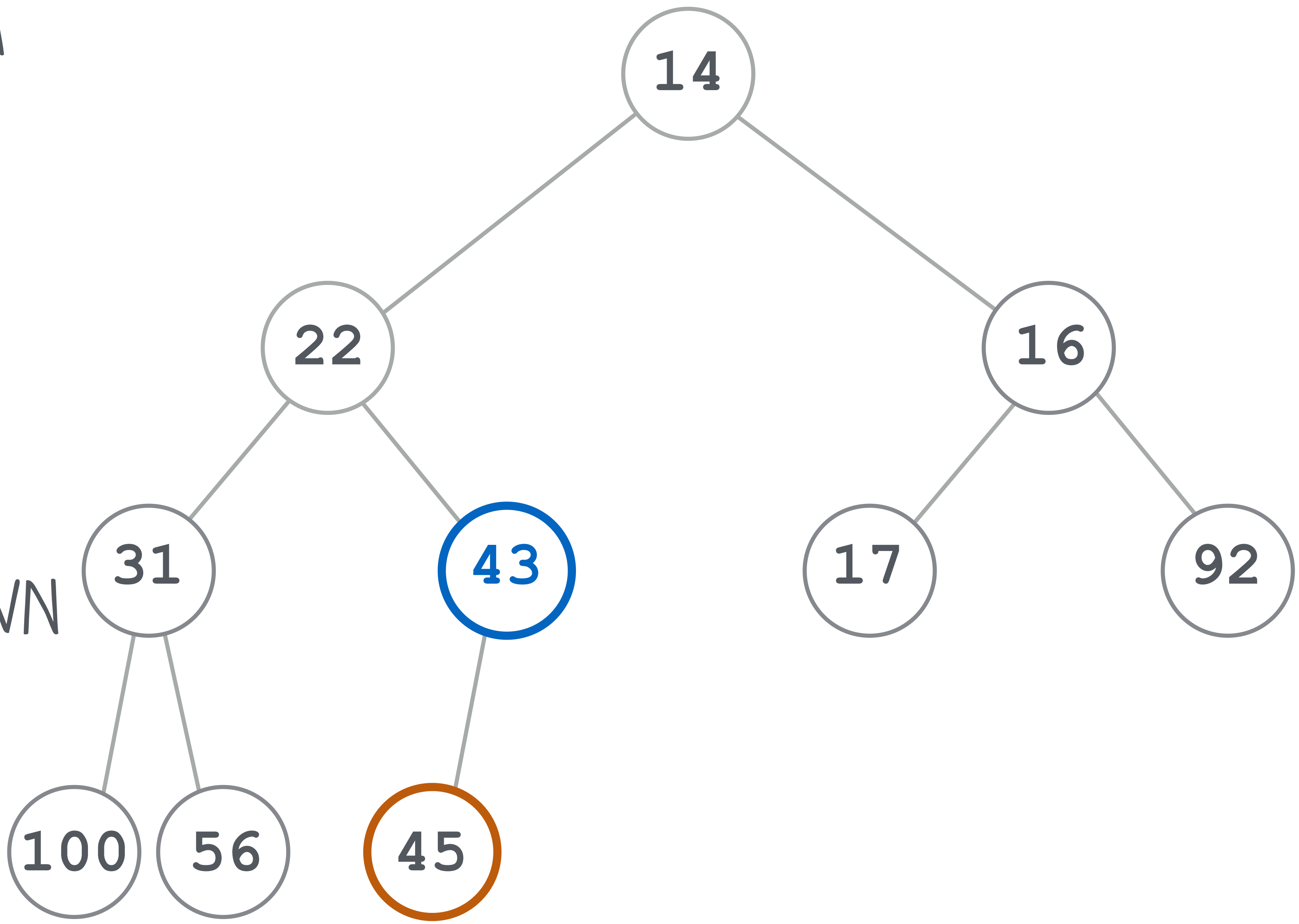

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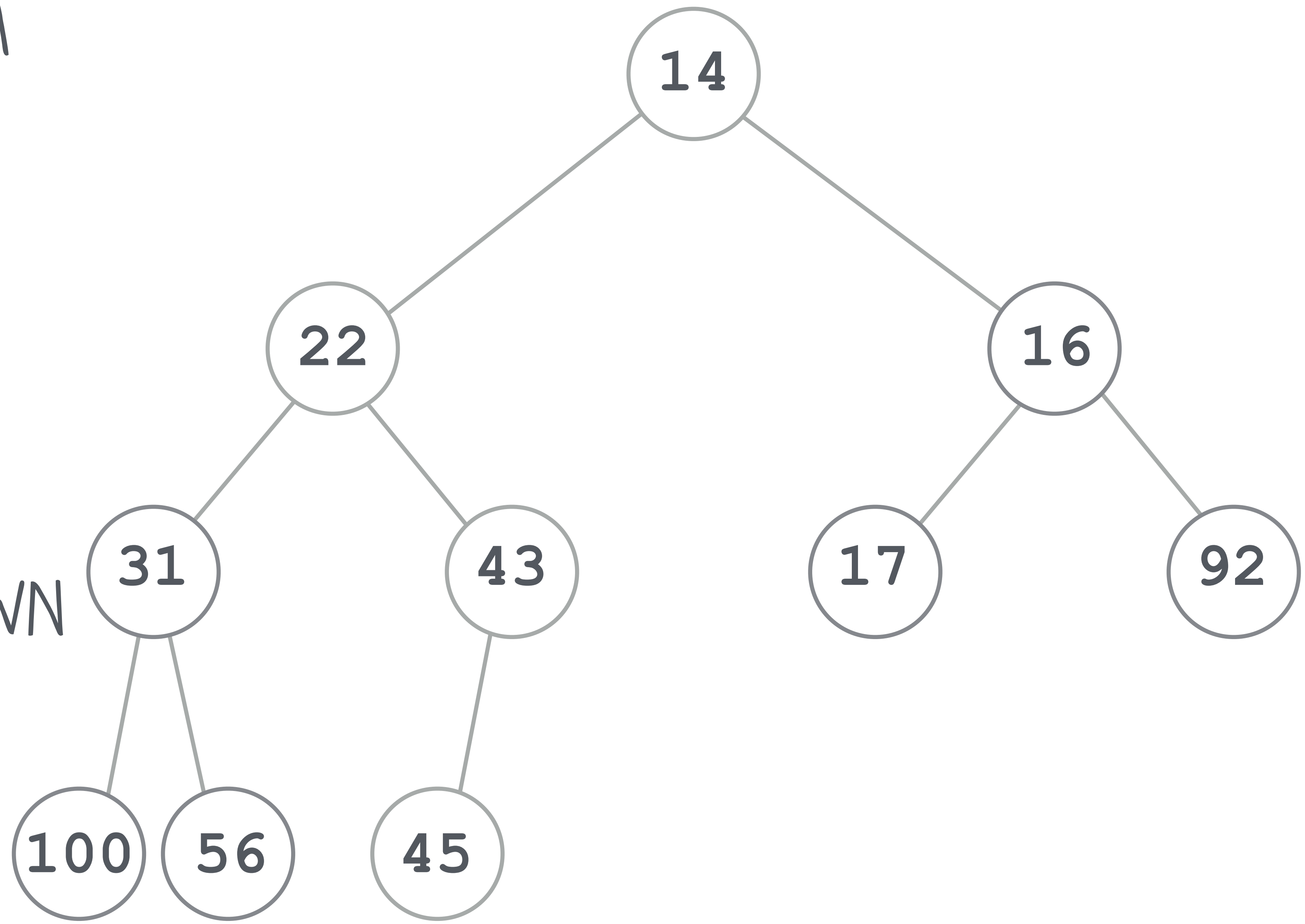

cost of remove

- -worst case is **O(logN)**
  - -percolating down to the bottom level

- -average case is also **O(logN)**
  - -rarely terminates more than 1-2 levels from the bottom… why?

recap

-priority queues can be implemented any number of ways

-a binary heap's main use is for implementing priority queues

-remember, the basic priority queue operations are:

- `-add`
- `-findMin`
- `-deleteMin`

-the average cases for a PQ implemented with a binary heap:

- -add
  - -**O(1):** *percolate up (average of 2.6 compares)*
- -findMin
  - - **O(1):** *just return the root*
- -deleteMin
  - - **O(logN):** *percolate down (rarely terminates before near the bottom of the tree)*

next time…

- reading
    - chapter 21 in book

- homework
    - assignment 10 due Thursday